One Predictor Linear Regression Model
(Population Model)

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

Score on Y for the ith individual = Y Intercept + [Slope (Effect) × Score on X for the ith individual] + Error

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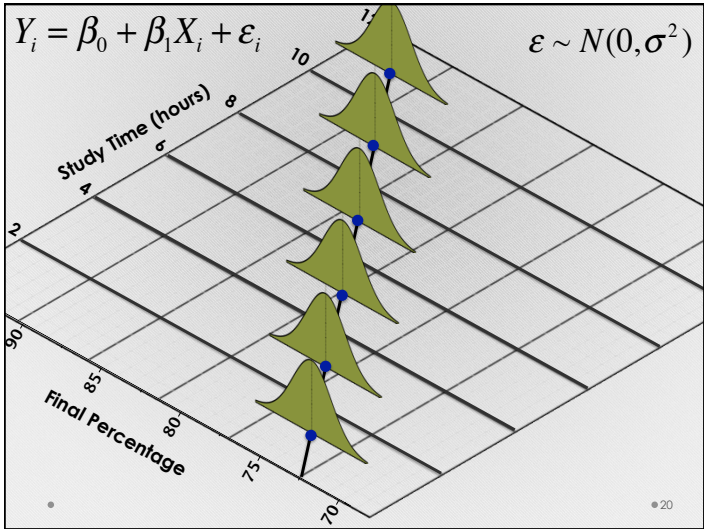

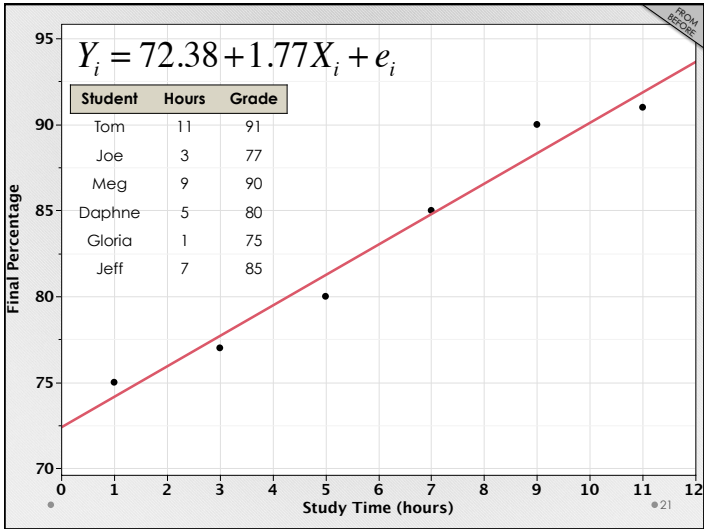

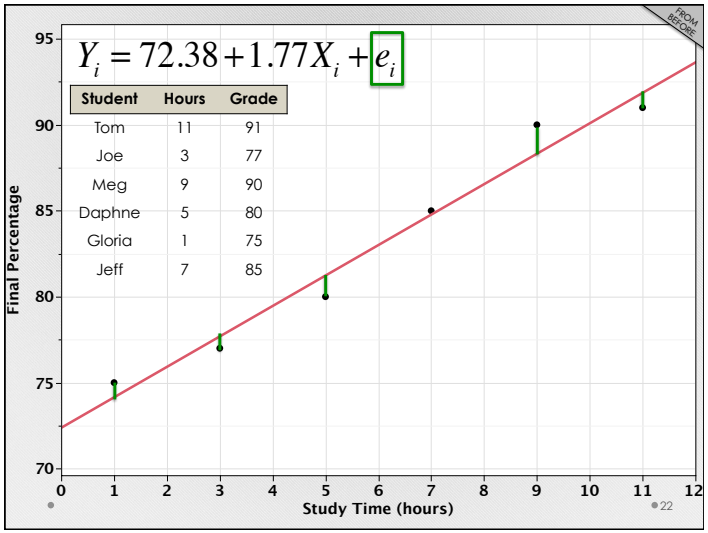

Estimation of σ^2

$$MS_{error} = \frac{SS_{error}}{n-2} \qquad MS_{error} \hat{=} \sigma^2$$

$$SS_{\text{error}} = \sum (Y_i - \hat{Y}_i)^2 = \sum e_i^2$$

23

Statistical Inferences with Regression

- **Tests of Parameters:**
  - **Test of b_1**
    In the population is the regression slope different from 0?
  - **Test of b_0**
    In the population is the Y-intercept different from 0?
- **Prediction:**
  - **Confidence Interval for the Conditional Mean of Y at X_i**
    Where do we believe the mean is for all people with a particular level of X (point estimate ± interval)
  - **Confidence Interval for Y for an Individual at X_i**
    Where do we believe the score is for a particular person at a particular level of X (point estimate ± interval)

24

Statistical Inferences with Regression

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25

Approaches to Statistical Inference with Regression

Sampling Distributions Known (T-Tests)

Analysis of Variance & General Linear Test (F-Tests)

26

Approaches to Statistical Inference with Regression

Sampling Distributions Known (T-Tests)

Analysis of Variance & General Linear Test (F-Tests)

27

T-Statistic, General Form

FROM BEFORE

$$t = \frac{\text{sample statistic - population parameter}}{\text{estimated standard error of statistic}}$$

28

T-Test Statistic for One Population Mean

FROM BEFORE

$$t_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}}$$

Value if H_0 = True

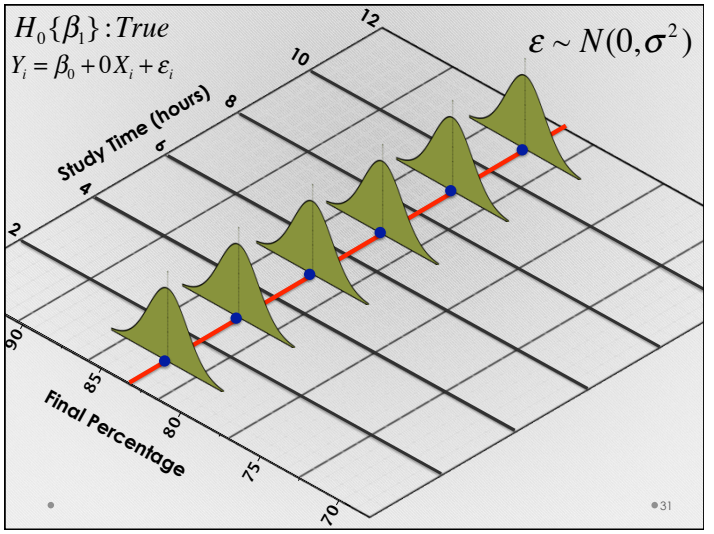
29

T-Test for β_1

$$t_{b_1} = \frac{b_1 - \beta_1}{s_{b_1}}$$

Value if H_0 = True

30



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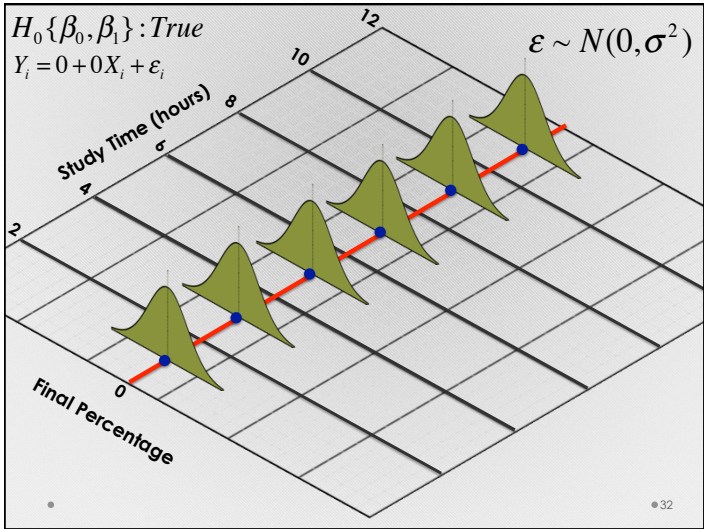


32

T-Test for β_1

$$t_{b_1} = \frac{b_1 - \beta_1}{s_{b_1}}$$

Value if H_0 = True

33

T-Test for β_1

$$t_{b_1} = \frac{b_1 - 0}{s_{b_1}}$$

Value if H_0 = True

T-Test for β_1

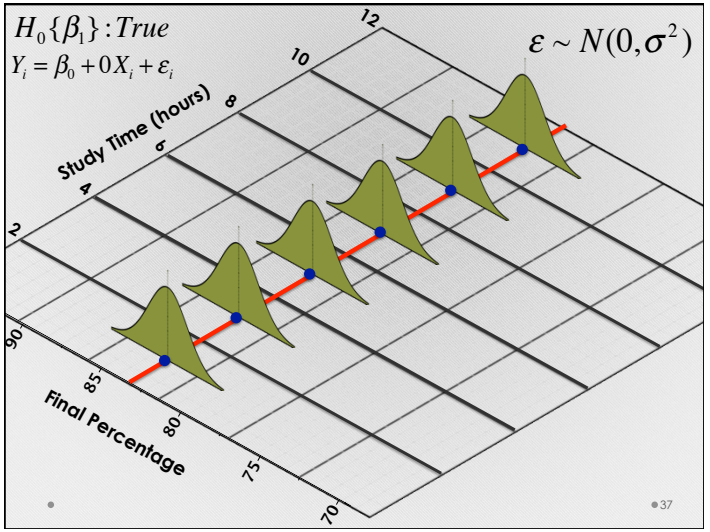
$$t_{b_1} = \frac{b_1}{s_{b_1}}$$

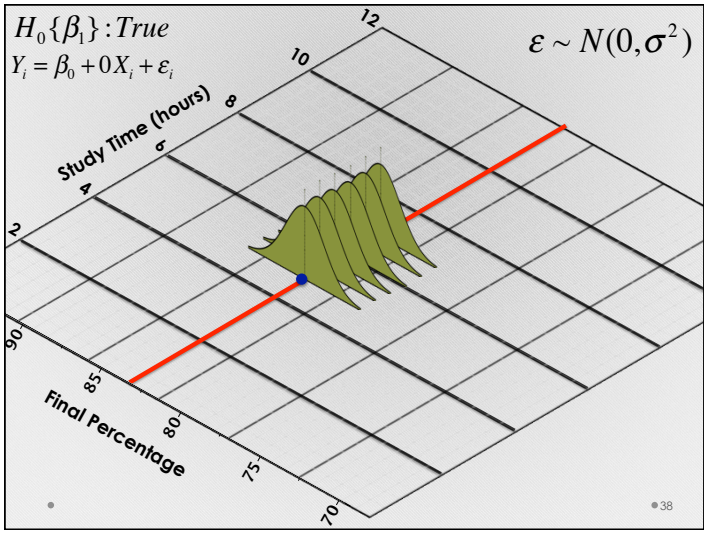
T-Test for β_1

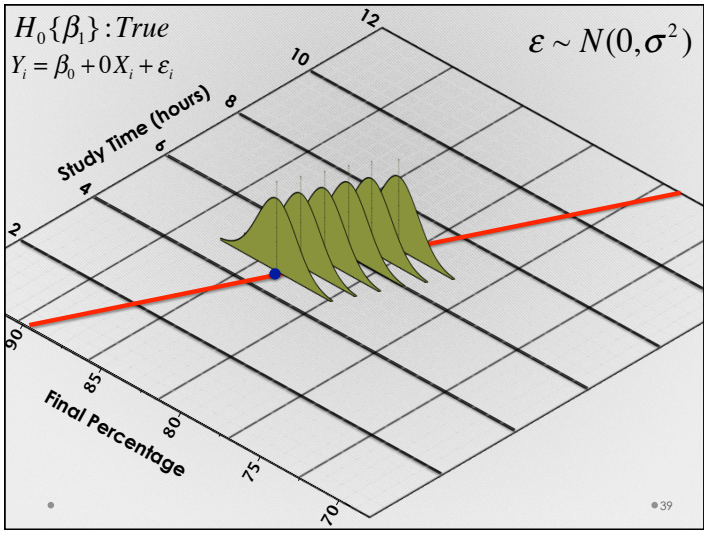
$$t_{b_1} = \frac{b_1}{s_{b_1}}$$

$$s_{b_1} = \sqrt{\frac{MS_{error}}{SS_X}}$$

A function of the number of observations **AND** the spacing of observations




$H_0\{\beta_1\}: True$
$Y_i = \beta_0 + 0X_i + \varepsilon_i$
$\varepsilon \sim N(0,\sigma^2)$
Study Time (hours)
2
4
6
8
10
12
Final Percentage
90
85
80
75
70
38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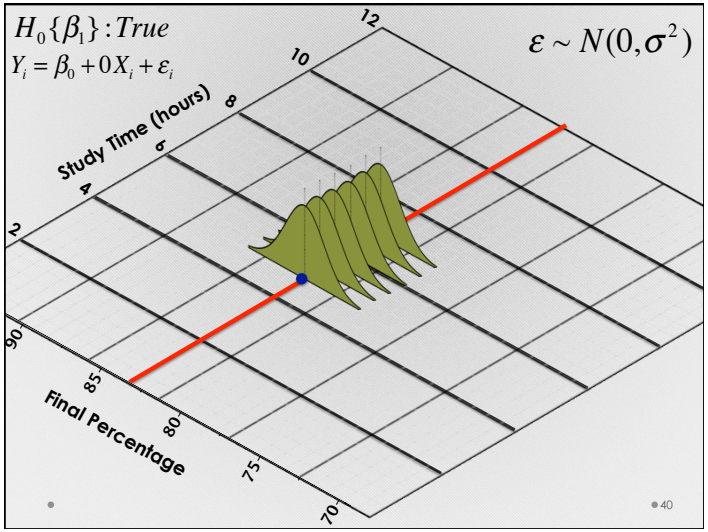
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Final Percentage
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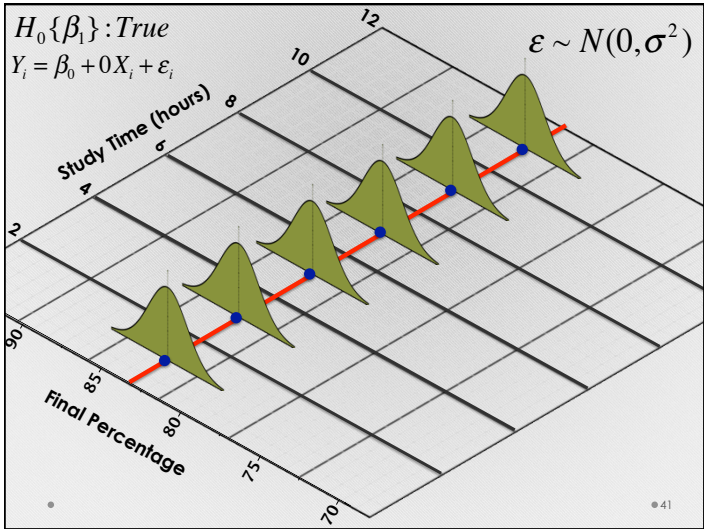
40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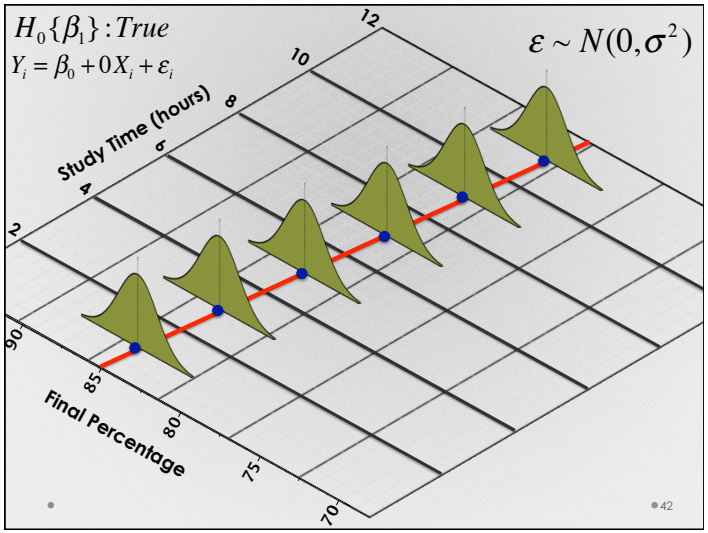
70
41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Study Time (hours)
2
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Final Percentage
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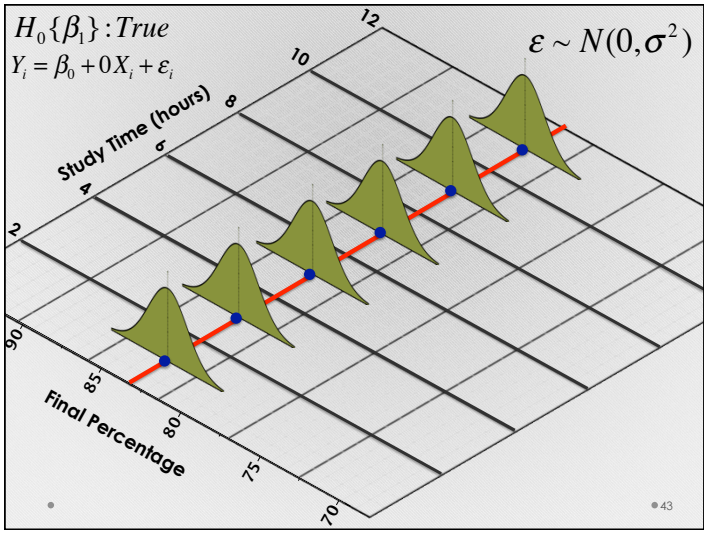
42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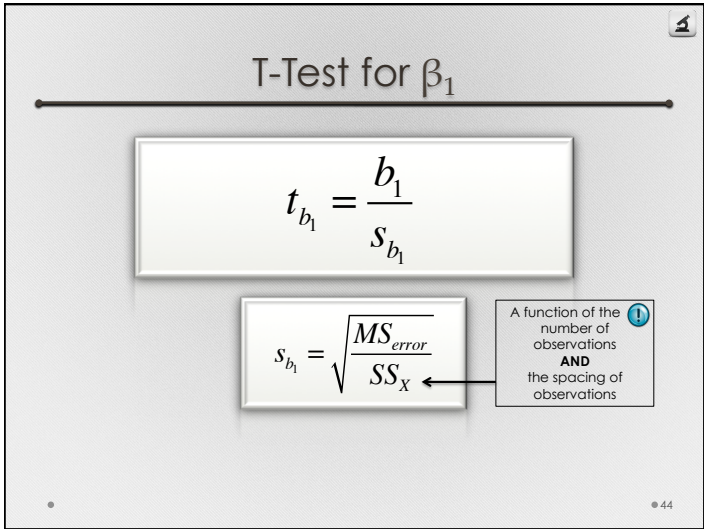
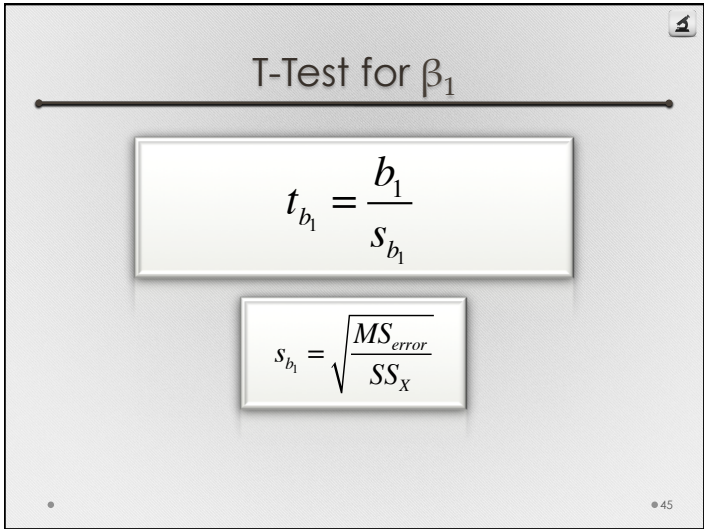

T-Test for β_1
$t_{b_1} = \frac{b_1}{s_{b_1}}$
$s_{b_1} = \sqrt{\frac{MS_{error}}{SS_X}}$
A function of the number of observations AND the spacing of observations
44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45

T-Test for β_0

$$t_{b_0} = \frac{b_0 - \beta_0}{s_{b_0}}$$

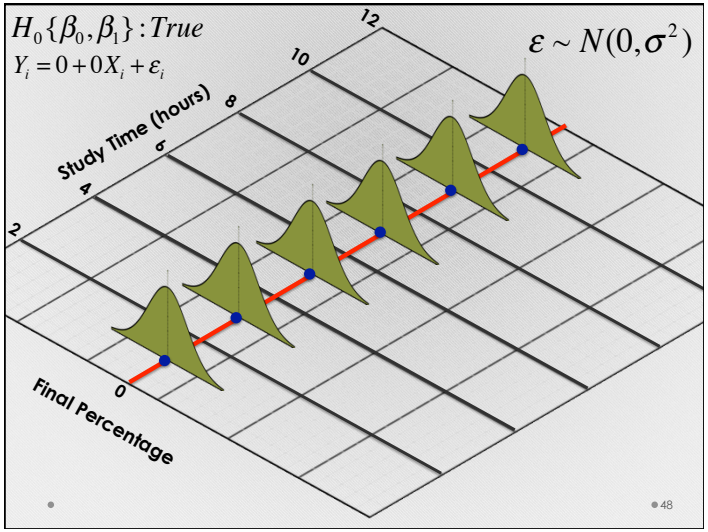
46

T-Test for β_0

$$t_{b_0} = \frac{b_0 - \beta_0}{s_{b_0}}$$

Value if H_0 = True

47



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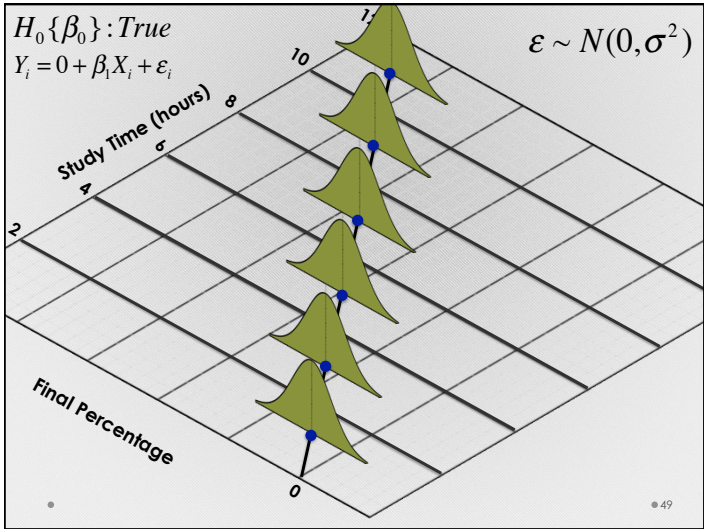


49

T-Test for β_0

$$t_{b_0} = \frac{b_0 - \beta_0}{s_{b_0}}$$

Value if H_0 = True

50

T-Test for β_0

$$t_{b_0} = \frac{b_0 - 0}{s_{b_0}}$$

Value if H_0 = True

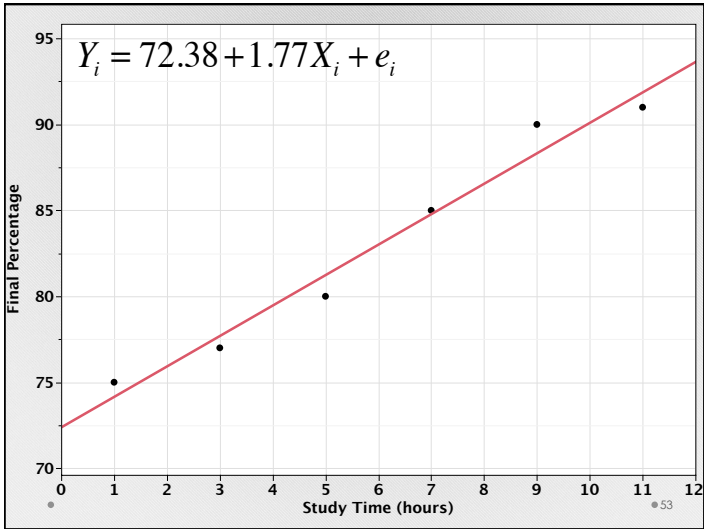
51

T-Test for β_0

$$t_{b_0} = \frac{b_0}{s_{b_0}}$$

$$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$$

52



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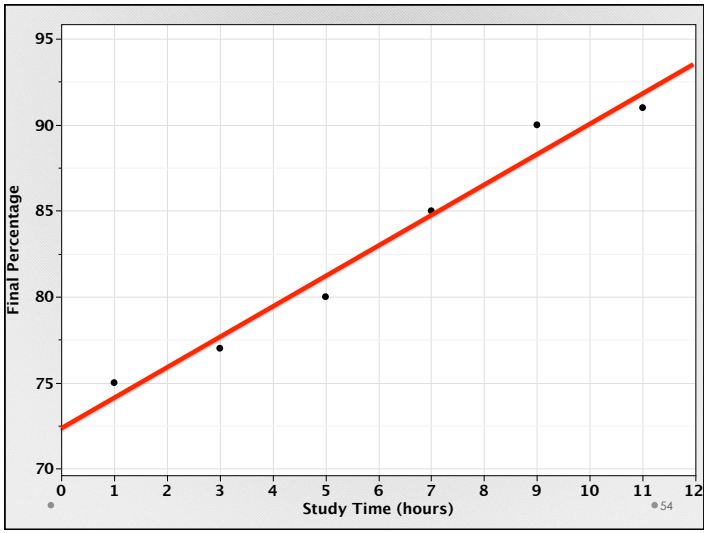
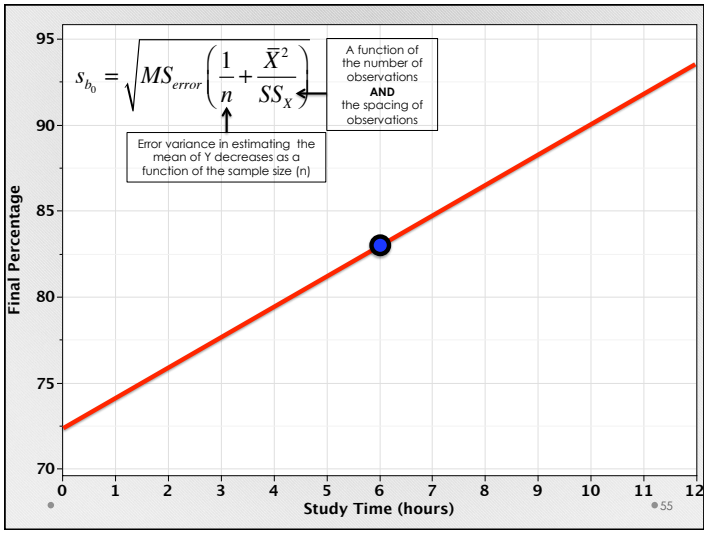
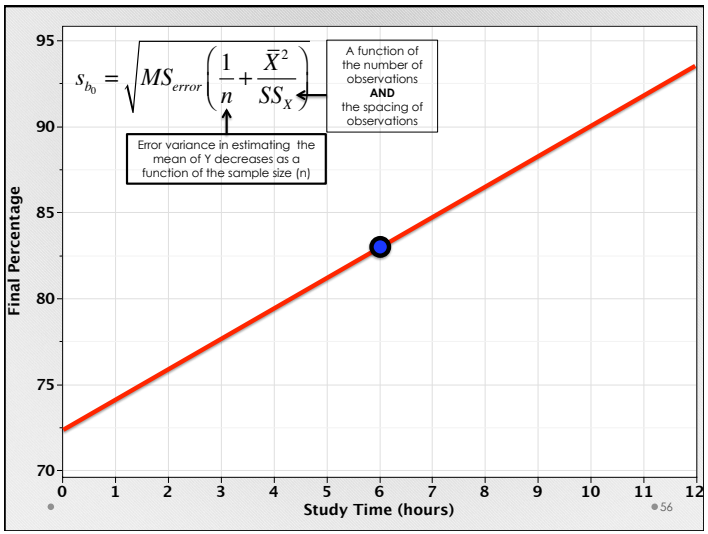

Final Percentage
Study Time (hours)
54


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
A function of the number of observations AND the spacing of observations
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
Final Percentage
Study Time (hours)
55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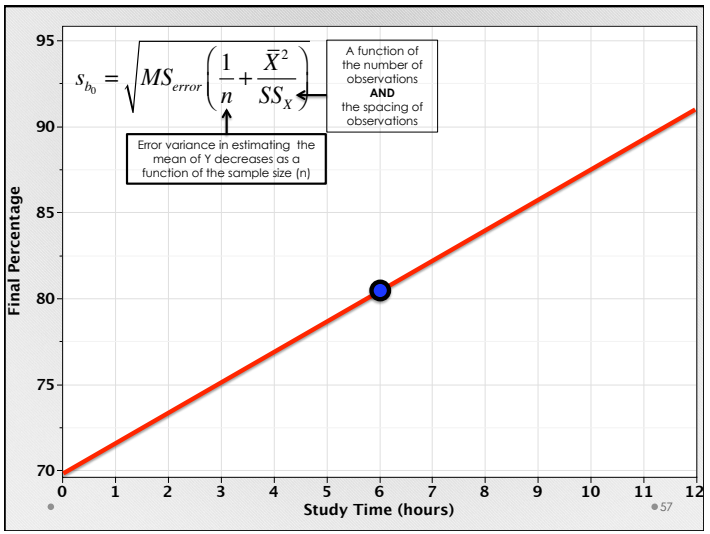
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Final Percentage
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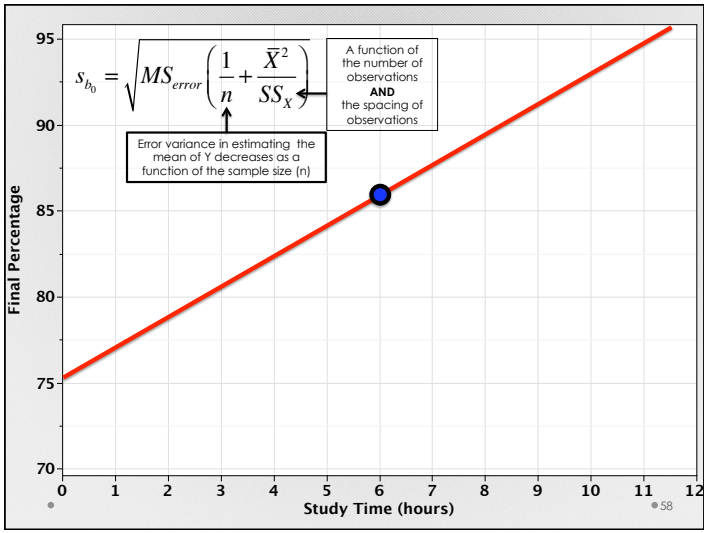
57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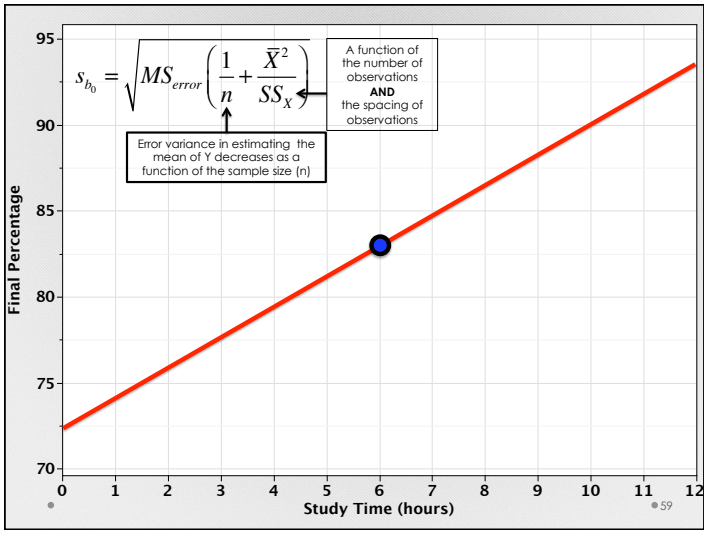
Final Percentage
Study Time (hours)
58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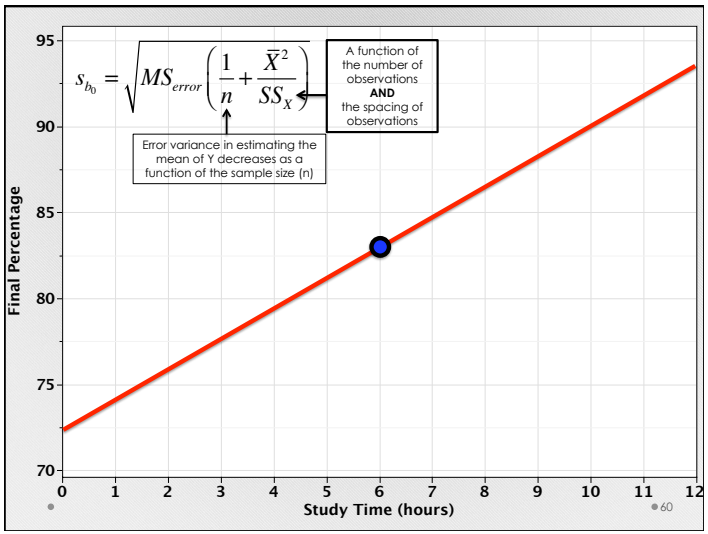
59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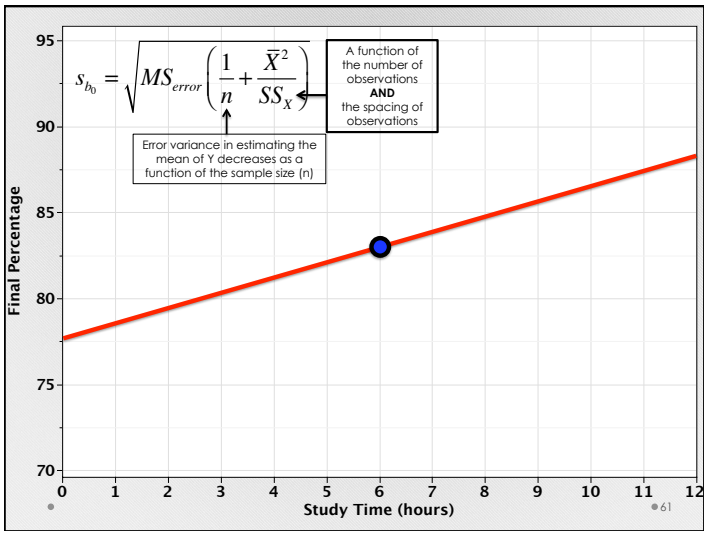
60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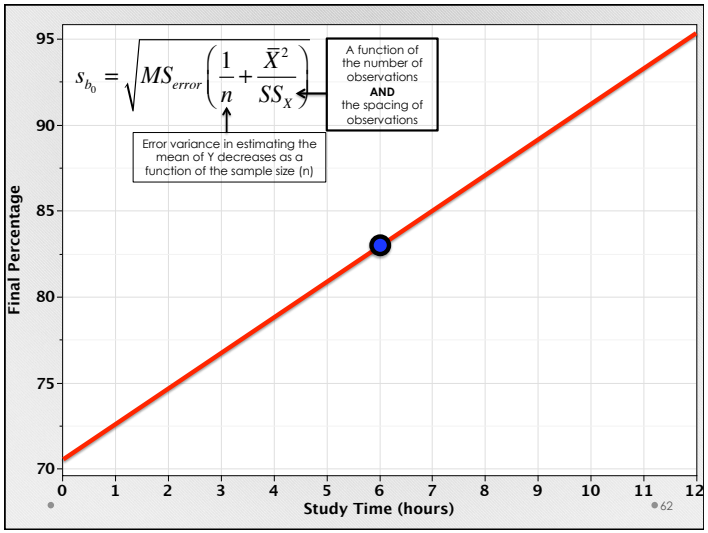
Final Percentage
Study Time (hours)
61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Final Percentage
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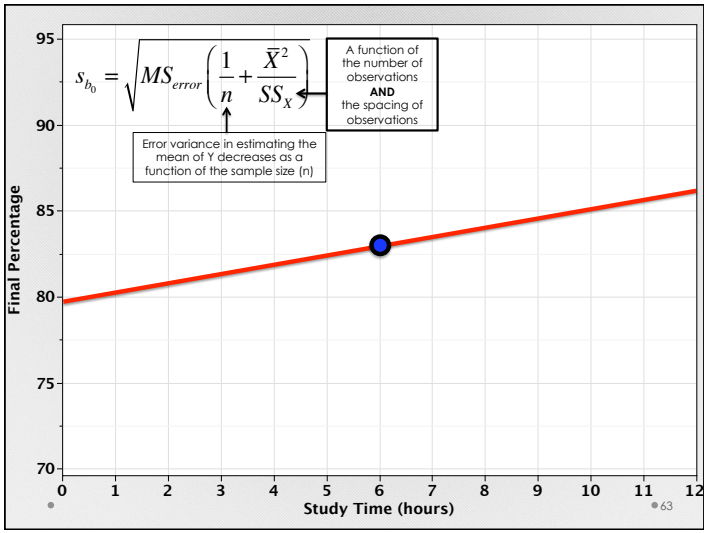
62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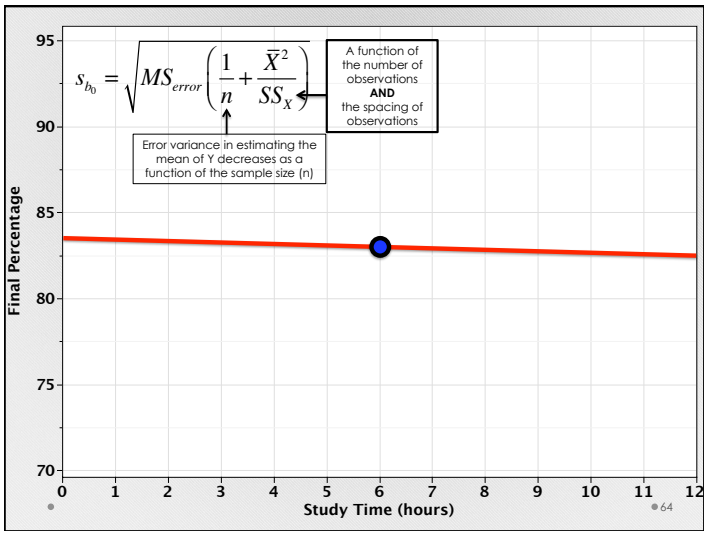
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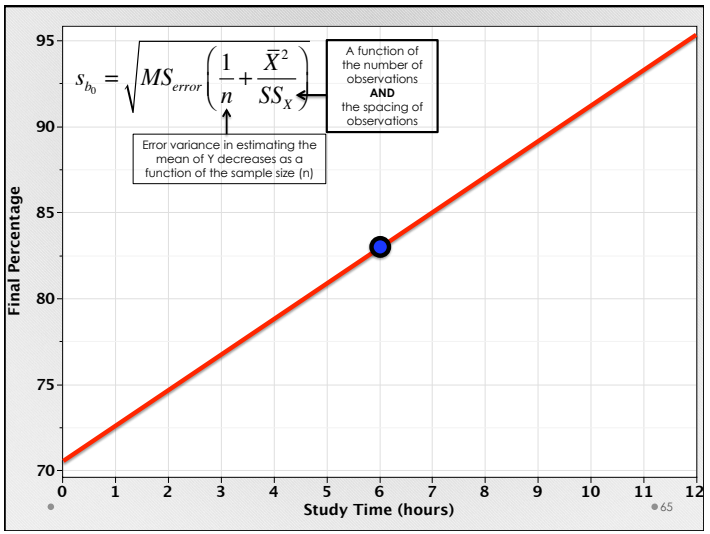
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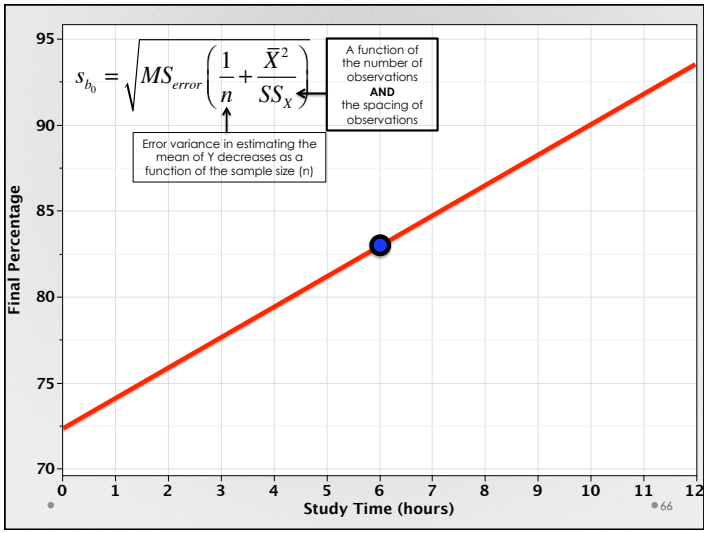
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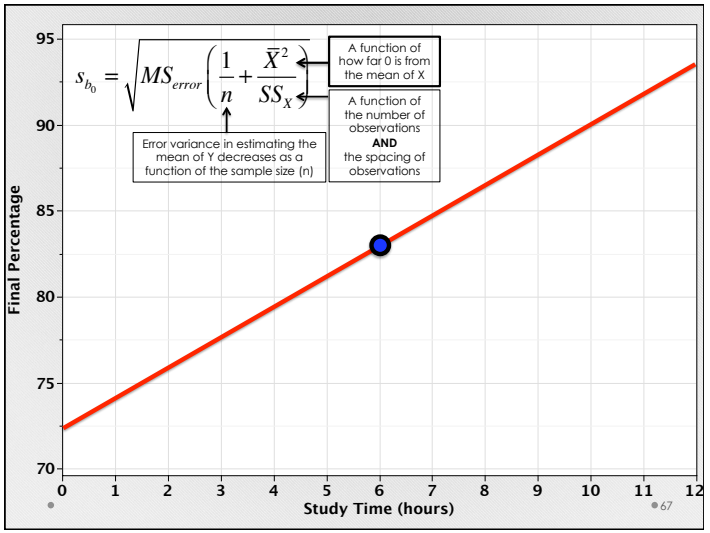
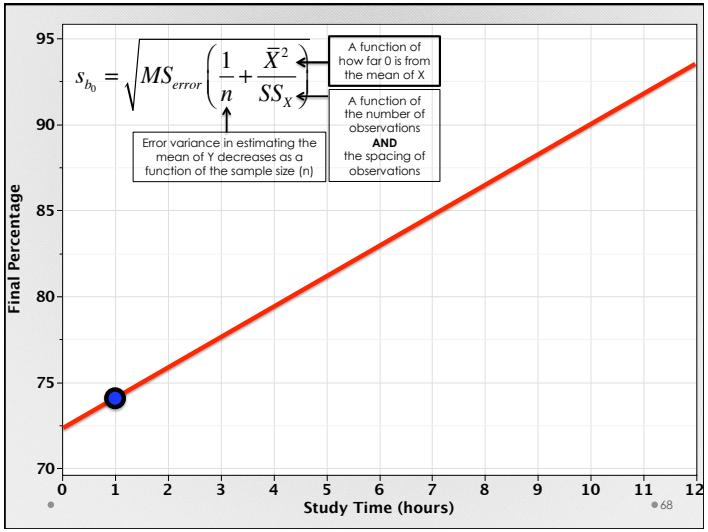

$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
Final Percentage
Study Time (hours)
67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Error variance in estimating the mean of Y decreases as a function of the sample size (n)
Final Percentage
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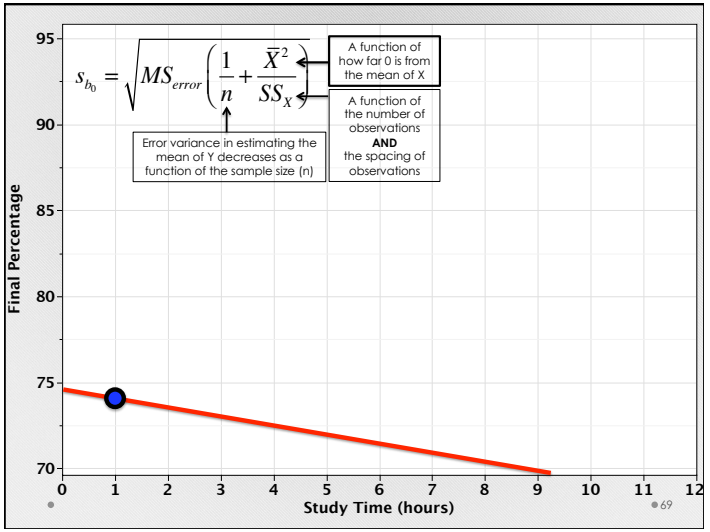
68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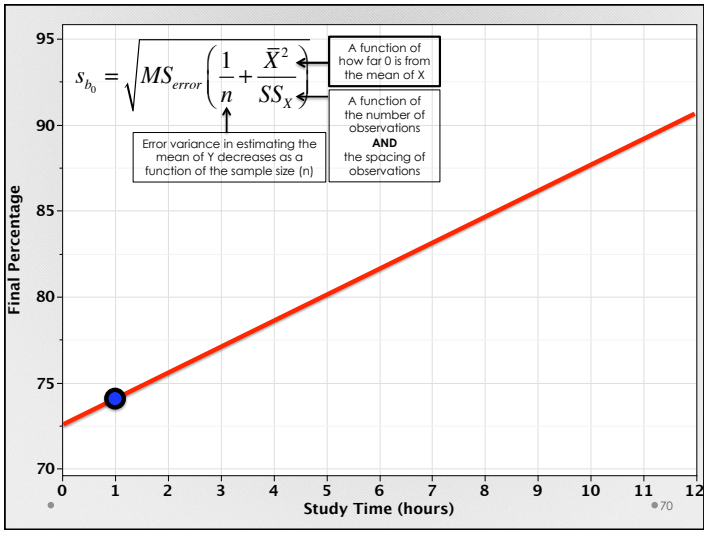
69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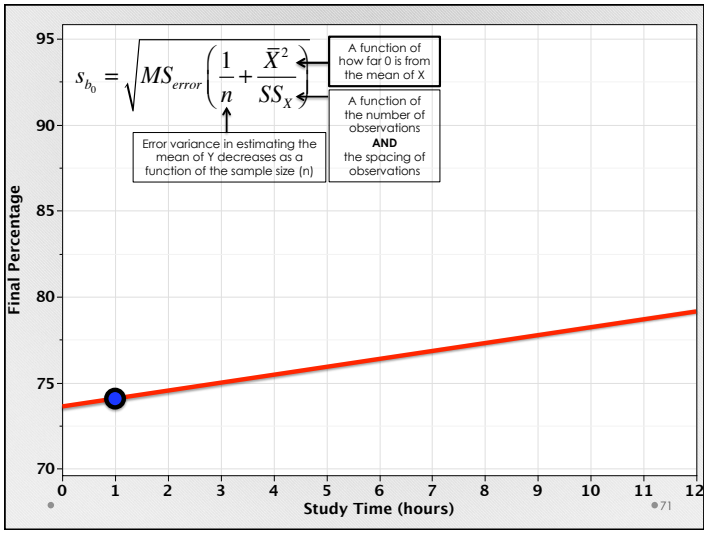
70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Error variance in estimating the mean of Y decreases as a function of the sample size (n)
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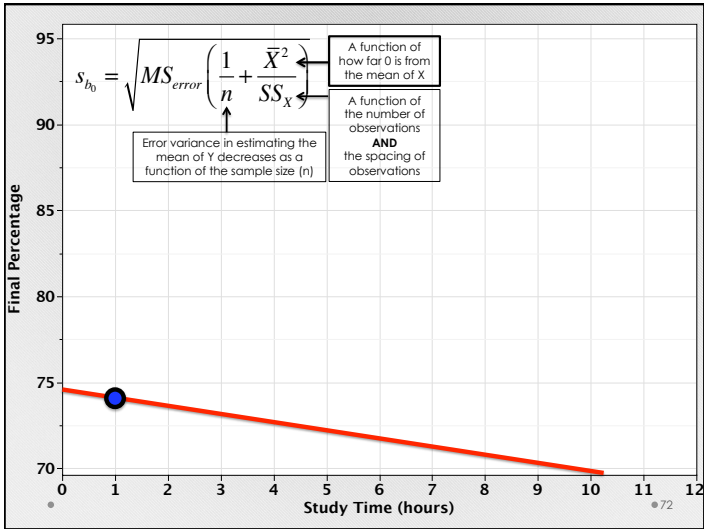
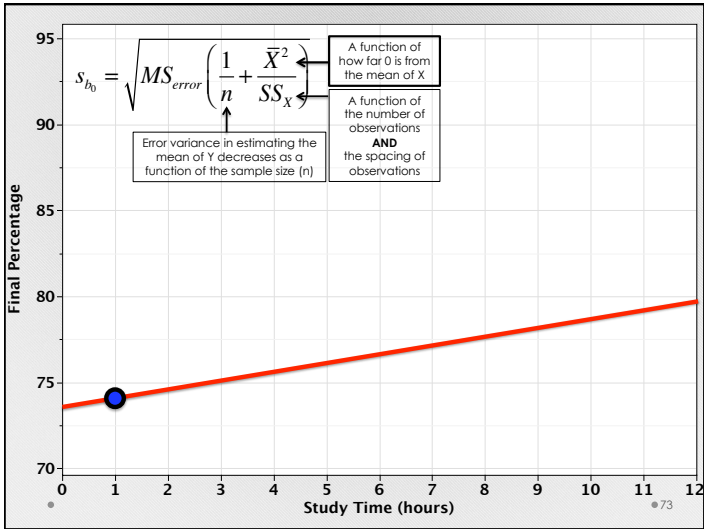
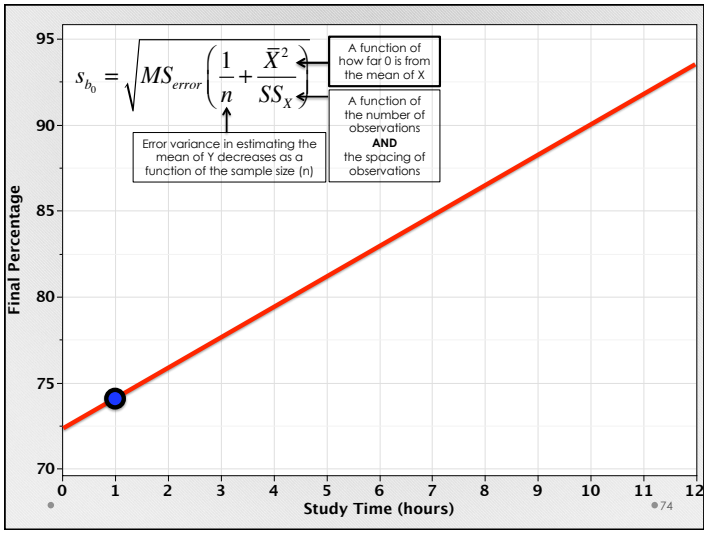
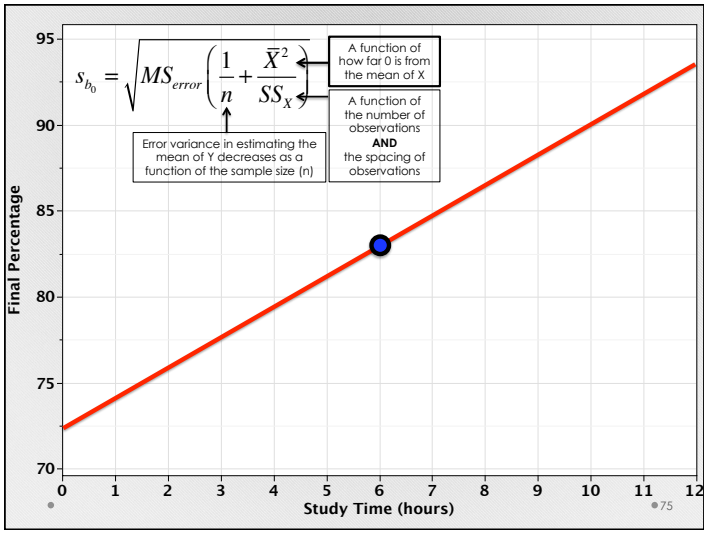
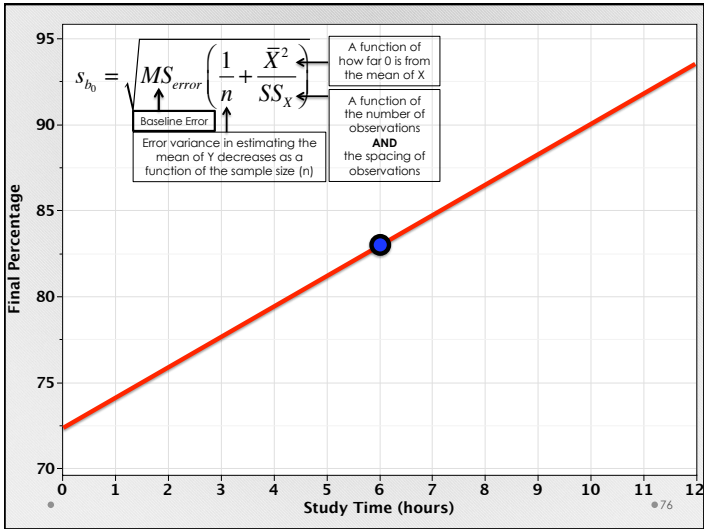
Study Time (hours)
71

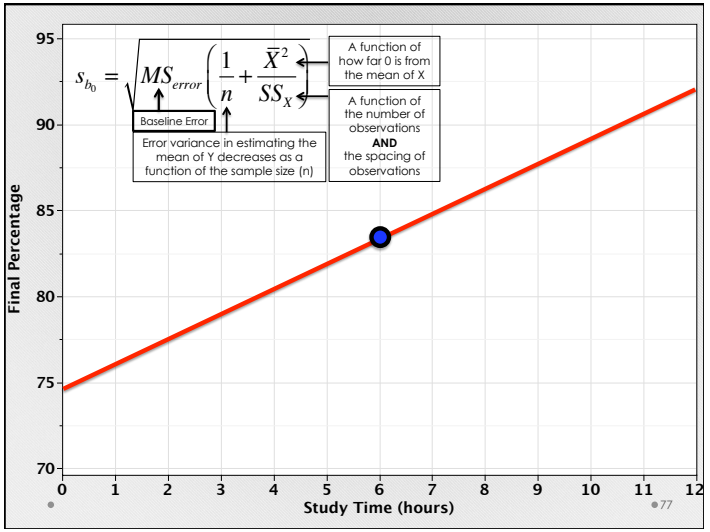

$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
Final Percentage
Study Time (hours)
72


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
Final Percentage
Study Time (hours)
73


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
Final Percentage
Study Time (hours)
74


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
Final Percentage
Study Time (hours)
75


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
Baseline Error
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
Final Percentage
Study Time (hours)
76


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
Baseline Error
A function of how far 0 is from the mean of X
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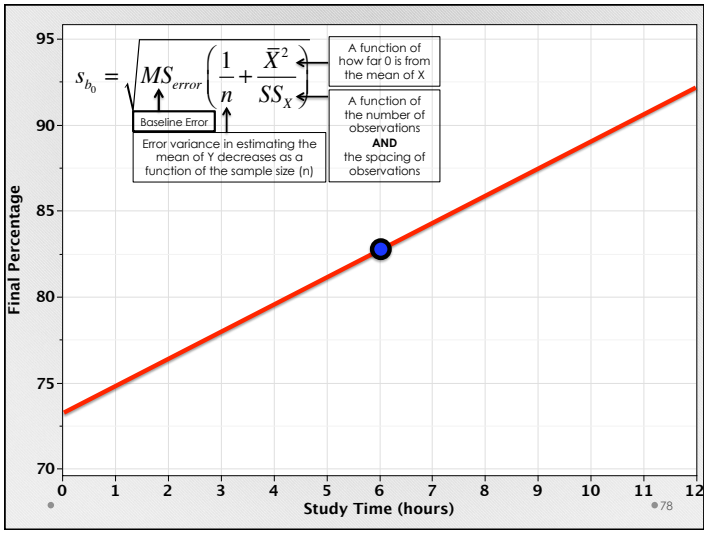
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
Final Percentage
Study Time (hours)
77


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
Baseline Error
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
A function of how far 0 is from the mean of X
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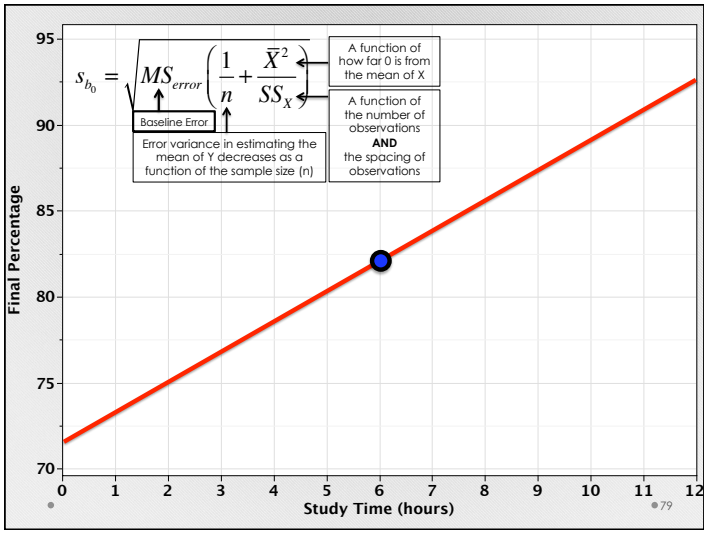
Final Percentage
Study Time (hours)
78


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
Baseline Error
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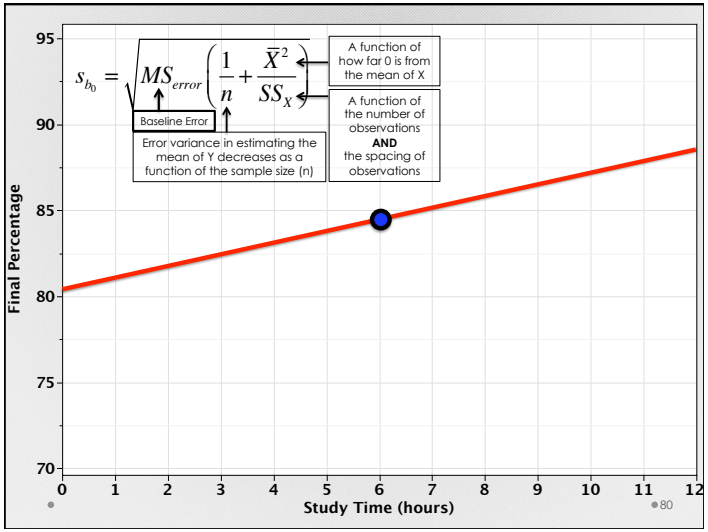
79


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
Baseline Error
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
A function of how far 0 is from the mean of X
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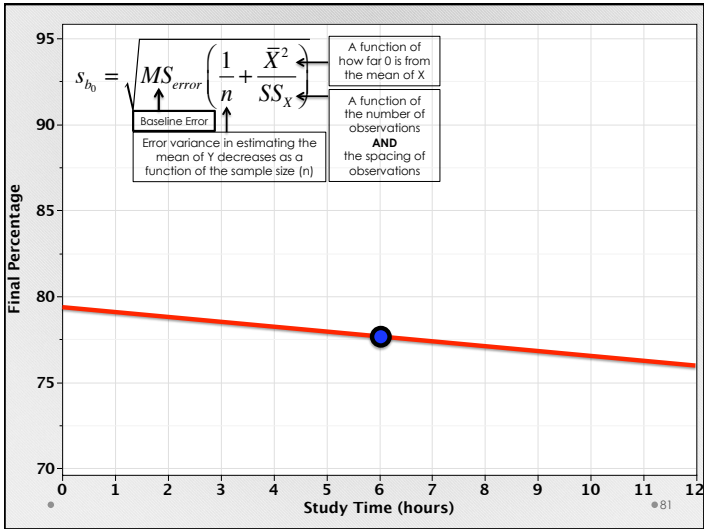
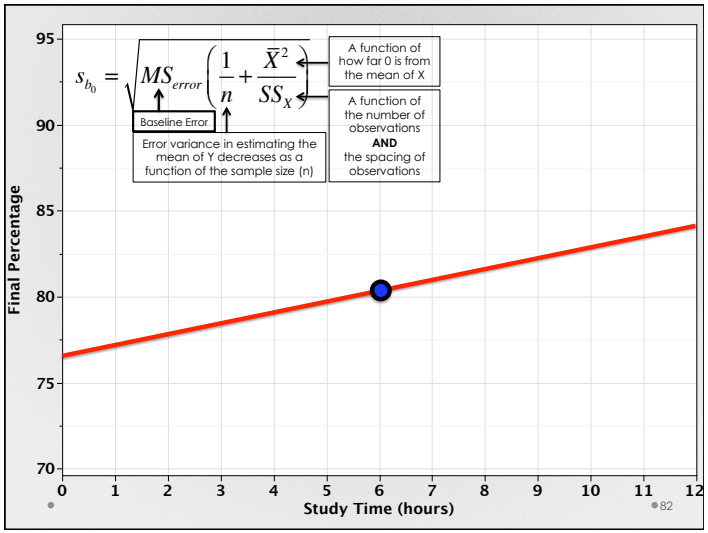
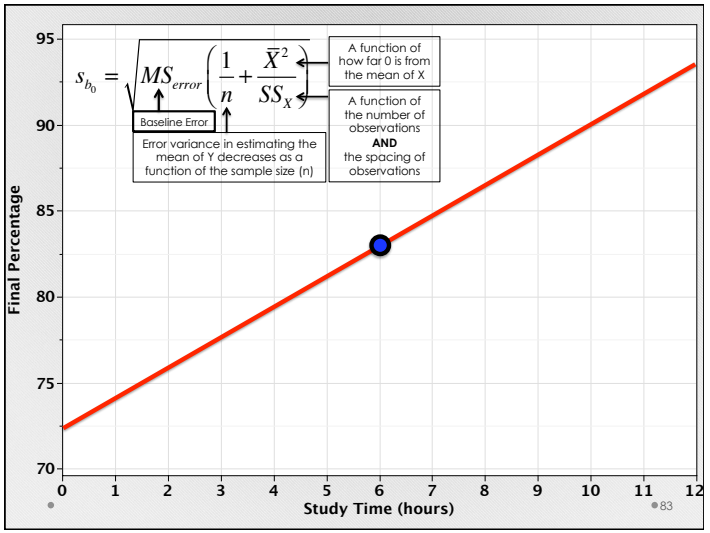
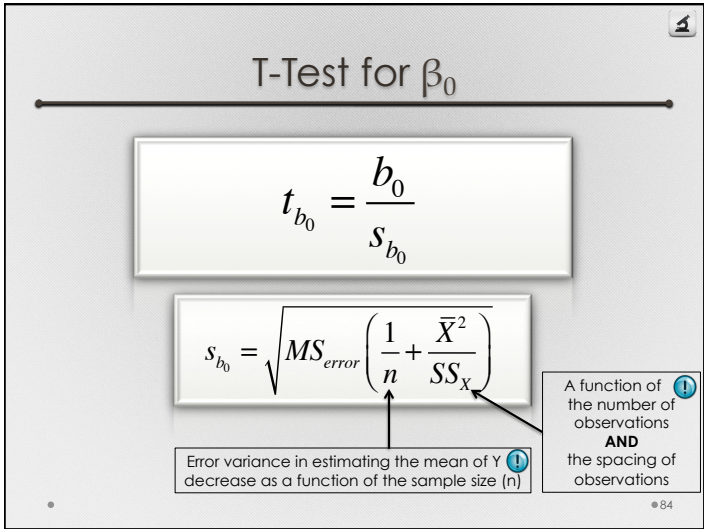
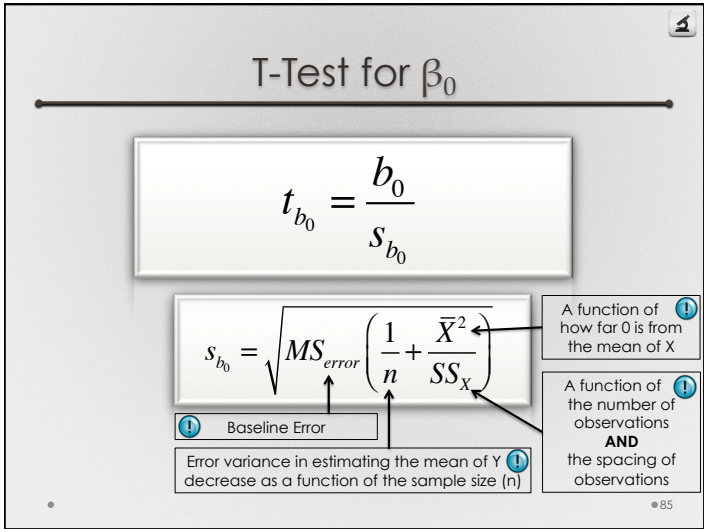
Final Percentage
Study Time (hours)
80


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
Baseline Error
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Final Percentage
Study Time (hours)
81


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
Baseline Error
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Final Percentage
Study Time (hours)
82


$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
Baseline Error
Error variance in estimating the mean of Y decreases as a function of the sample size (n)
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Final Percentage
Study Time (hours)
83


T-Test for β_0
$t_{b_0} = \frac{b_0}{s_{b_0}}$
$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
Error variance in estimating the mean of Y decrease as a function of the sample size (n)
A function of the number of observations AND the spacing of observations
84


T-Test for β_0
$t_{b_0} = \frac{b_0}{s_{b_0}}$
$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$
A function of how far 0 is from the mean of X
A function of the number of observations AND the spacing of observations
Baseline Error
Error variance in estimating the mean of Y decrease as a function of the sample size (n)
85

T-Test for β_0

$$t_{b_0} = \frac{b_0}{s_{b_0}}$$

$$s_{b_0} = \sqrt{MS_{error}\left(\frac{1}{n} + \frac{\bar{X}^2}{SS_X}\right)}$$

86

Approaches to Statistical Inference with Regression

Sampling Distributions Known (T-Tests)

Analysis of Variance & General Linear Test (F-Tests)

87

Approaches to Statistical Inference with Regression

Sampling Distributions Known (T-Tests)

Analysis of Variance & General Linear Test (F-Tests)

88

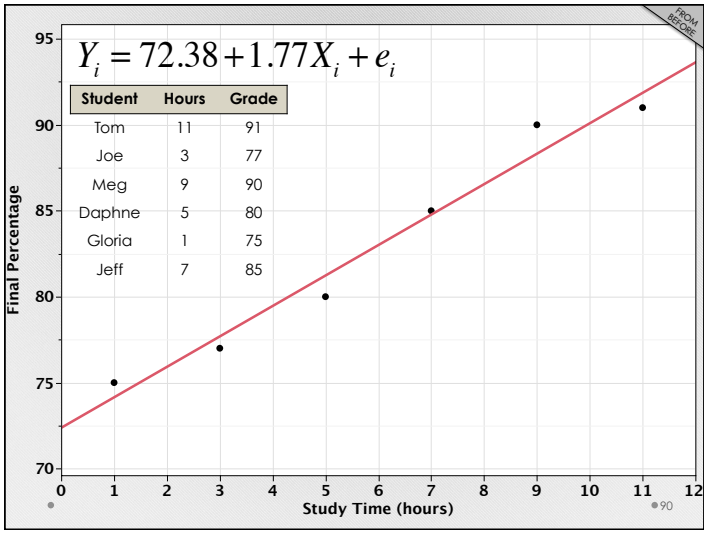
One Predictor Linear Regression Model (Population Model)

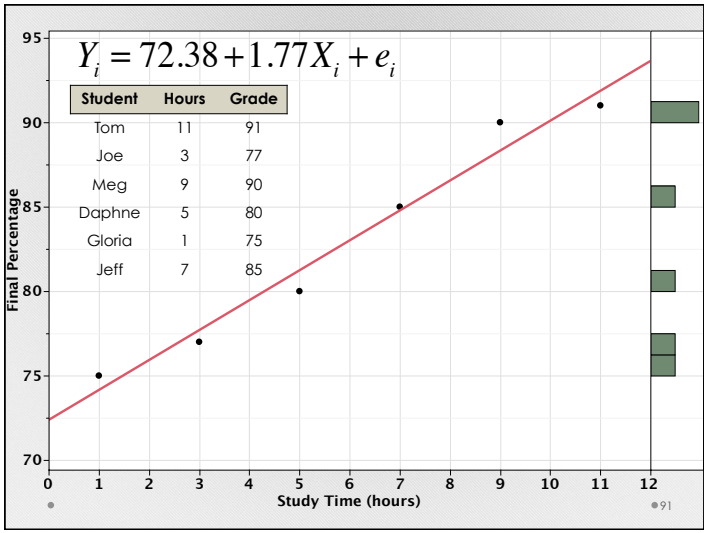
FROM BEFORE

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

Score on Y for the ith individual = Y Intercept + Slope (Effect) × Score on X for the ith individual + Error

89


$Y_i = 72.38 + 1.77X_i + e_i$
Student Hours Grade
Tom 11 91
Joe 3 77
Meg 9 90
Daphne 5 80
Gloria 1 75
Jeff 7 85
Final Percentage
Study Time (hours)
90


$Y_i = 72.38 + 1.77X_i + e_i$
Student Hours Grade
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Final Percentage
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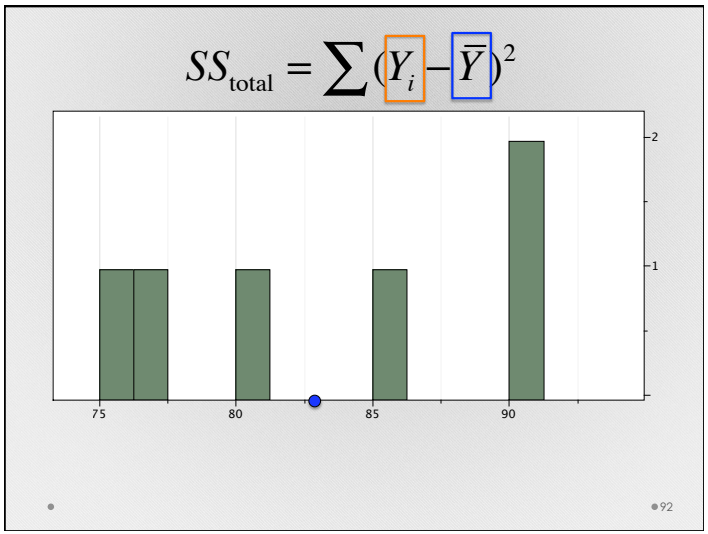
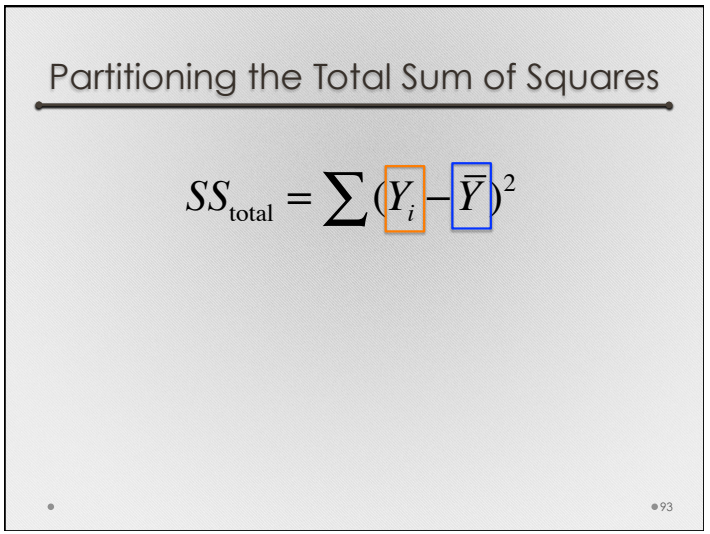
91


$SS_{\text{total}} = \sum (Y_i - \bar{Y})^2$
92


Partitioning the Total Sum of Squares
$SS_{\text{total}} = \sum (Y_i - \bar{Y})^2$
93

Partitioning the Total Sum of Squares

$$SS_{\text{total}} = \sum (Y_i - \bar{Y})^2$$

Partitioning the Total Sum of Squares

$$SS_{\text{total}} = \sum (Y_i - \bar{Y})^2$$

$$= SS_{\text{error}} = \sum (Y_i - \hat{Y}_i)^2$$

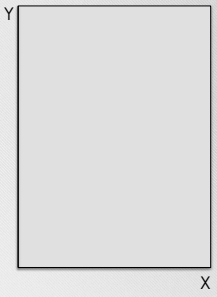
Partitioning the Total Sum of Squares

$$SS_{\text{total}} = \sum (Y_i - \bar{Y})^2$$

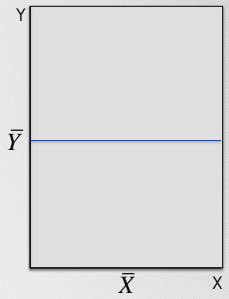
$$= SS_{\text{error}} = \sum (Y_i - \hat{Y}_i)^2$$

$$+ \quad SS_{\text{regression}} = \sum (\hat{Y}_i - \bar{Y})^2$$

$Y_i - \bar{Y}$
Total Deviation

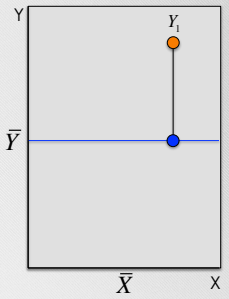


$Y_i - \bar{Y}$
Total Deviation



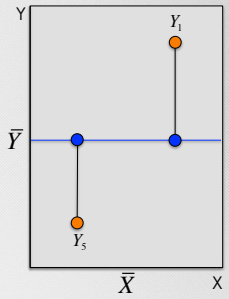
98

$Y_i - \bar{Y}$
Total Deviation



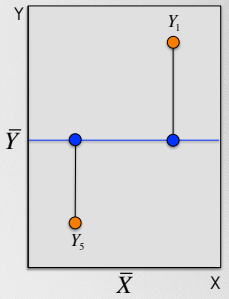
99

$Y_i - \bar{Y}$
Total Deviation



100

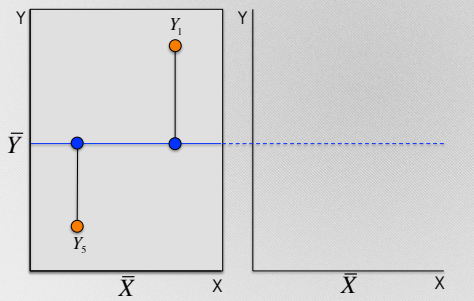
$Y_i - \bar{Y}$
Total Deviation



$SS_{\text{total}} = \sum (Y_i - \bar{Y})^2$

101

$Y_i - \bar{Y}$
Total Deviation

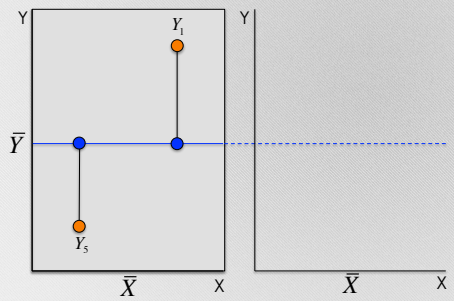


$SS_{total} = \sum (Y_i - \bar{Y})^2$

102

$Y_i - \bar{Y}$
Total Deviation

$Y_i - \hat{Y}_i$
Error Deviation

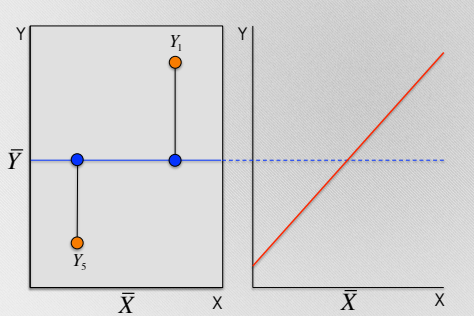


$SS_{total} = \sum (Y_i - \bar{Y})^2$ $SS_{error} = \sum (Y_i - \hat{Y}_i)^2$

103

$Y_i - \bar{Y}$
Total Deviation

$Y_i - \hat{Y}_i$
Error Deviation

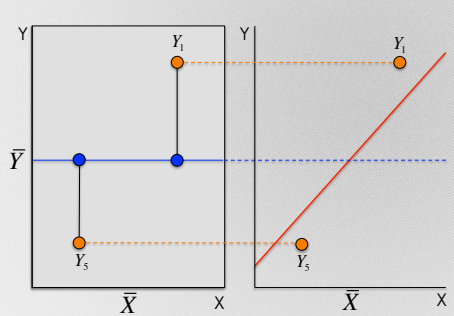


$SS_{total} = \sum (Y_i - \bar{Y})^2$ $SS_{error} = \sum (Y_i - \hat{Y}_i)^2$

104

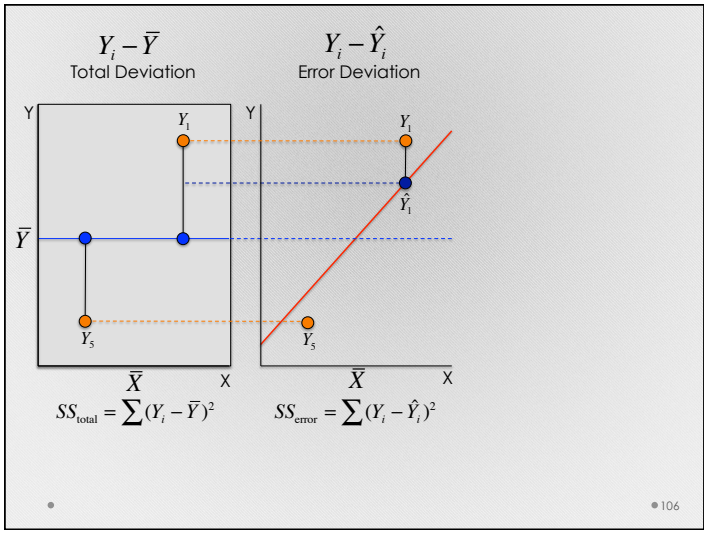
$Y_i - \bar{Y}$
Total Deviation

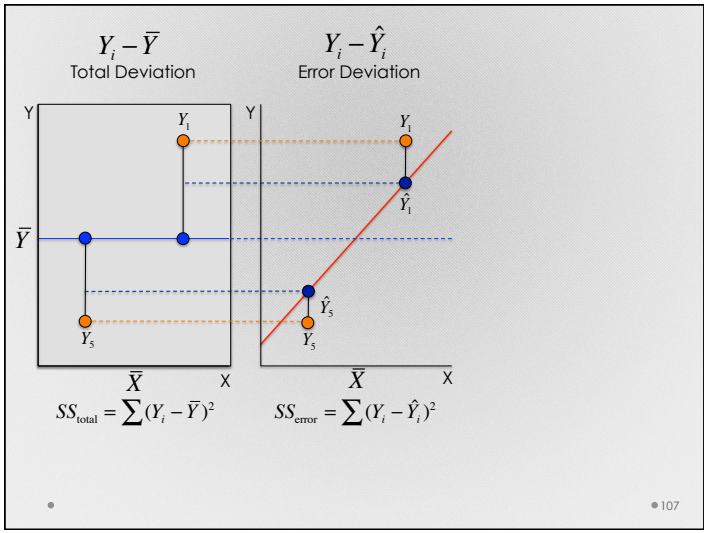
$Y_i - \hat{Y}_i$
Error Deviation

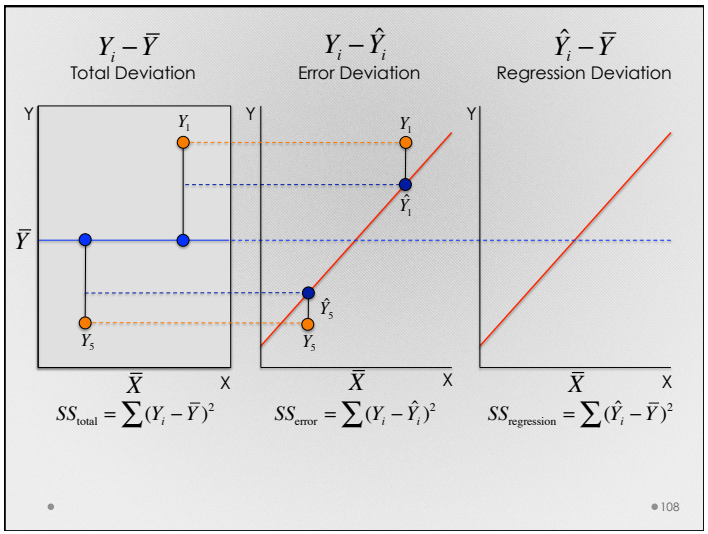


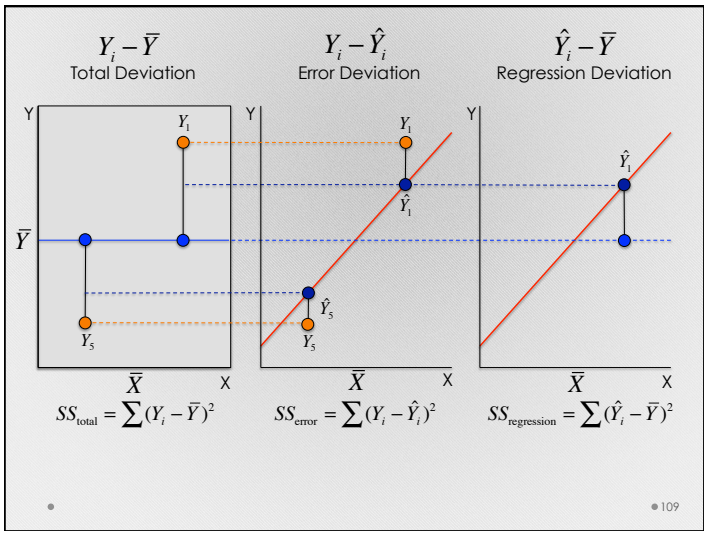
$SS_{total} = \sum (Y_i - \bar{Y})^2$ $SS_{error} = \sum (Y_i - \hat{Y}_i)^2$

105


$Y_i - \bar{Y}$
Total Deviation
$Y_i - \hat{Y}_i$
Error Deviation
Y
X
Y_1
$\hat{Y}_1$
$\bar{Y}$
Y_5
$\hat{Y}_5$
$\bar{X}$
$SS_{total} = \sum (Y_i - \bar{Y})^2$
$SS_{error} = \sum (Y_i - \hat{Y}_i)^2$
106


$Y_i - \bar{Y}$
Total Deviation
$Y_i - \hat{Y}_i$
Error Deviation
Y
X
Y_1
$\hat{Y}_1$
$\bar{Y}$
Y_5
$\hat{Y}_5$
$\bar{X}$
$SS_{total} = \sum (Y_i - \bar{Y})^2$
$SS_{error} = \sum (Y_i - \hat{Y}_i)^2$
107


$Y_i - \bar{Y}$
Total Deviation
$Y_i - \hat{Y}_i$
Error Deviation
$\hat{Y}_i - \bar{Y}$
Regression Deviation
Y
X
Y_1
$\hat{Y}_1$
$\bar{Y}$
Y_5
$\hat{Y}_5$
$\bar{X}$
$SS_{total} = \sum (Y_i - \bar{Y})^2$
$SS_{error} = \sum (Y_i - \hat{Y}_i)^2$
$SS_{regression} = \sum (\hat{Y}_i - \bar{Y})^2$
108


$Y_i - \bar{Y}$
Total Deviation
$Y_i - \hat{Y}_i$
Error Deviation
$\hat{Y}_i - \bar{Y}$
Regression Deviation
Y
X
Y_1
$\hat{Y}_1$
$\bar{Y}$
Y_5
$\hat{Y}_5$
$\bar{X}$
$SS_{total} = \sum (Y_i - \bar{Y})^2$
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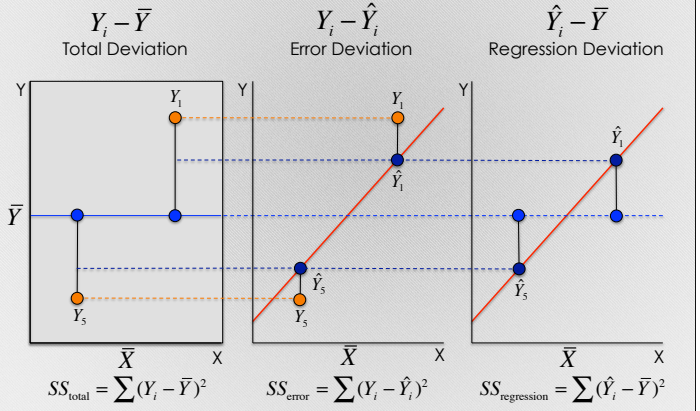
$SS_{regression} = \sum (\hat{Y}_i - \bar{Y})^2$
109



F-Test for Simple Regression

Test for Regression Slope

$$F = \frac{SS_{regression} / df_{regression}}{SS_{error} / df_{error}} = \frac{MS_{regression}}{MS_{error}}$$

Approaches to Statistical Inference with Regression

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Sampling Distributions Known (T-Tests)

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