

Experimental Designs with Repeated Measurements

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Repeated Measures Designs

A repeated measures design involves multiple observations from the same experimental unit under different conditions.

A factor with multiple levels measured for each unit is called a "Repeated factor" or "Within-Subject factor"

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	Wine 1	Wine 2	Wine 3	Wine 4
Judge 1				
Judge 2				
Judge 3				
Judge 4				
Judge 5				
Judge 6				
Judge 7				
Judge 8				
Judge 9				
...				

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GLM Distributional Assumptions

Error (in the population) is distributed $\varepsilon \sim N(0, \sigma^2)$ with mean = 0 and variance σ^2 normally and variance σ^2 i.i.d. Independent and Identically distributed

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Judge 9				
...				

Analytic Approaches for Repeated Measures Data

- General Linear Model ANOVA (EMS)
- Multivariate Analysis of Variance
- Linear Mixed Models (REML/ML)

One Repeated Factor Linear Model (Population Model)

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$Y_{ij} =$

Score on Y
of the i th individual in
the j th treatment

One Repeated Factor Linear Model (Population Model)

$$Y_{ij} = \mu_{..} +$$

Score on Y
of the i th individual in
the j th treatment = Grand
Mean +

One Repeated Factor Linear Model (Population Model)

$$Y_{ij} = \mu_{..} + \rho_i +$$

Score on Y
of the i th individual in
the j th treatment = Grand
Mean + Overall
offset for
Individual i +

One Repeated Factor Linear Model (Population Model)

$$Y_{ij} = \mu_{..} + \rho_i + \tau_j +$$

Score on Y
of the i th individual in
the j th treatment = Grand
Mean + Overall
offset for
Individual i + Treatment
offset for
level j

One Repeated Factor Linear Model (Population Model)

$$Y_{ij} = \mu_{..} + \rho_i + \tau_j + \epsilon_{ij}$$

Score on Y
of the i th individual in
the j th treatment = Grand
Mean + Overall
offset for
Individual i + Treatment
offset for
level j + Error

One Repeated Factor Linear Model (Population Model)

$$Y_{ij} = \mu_{..} + \rho_i + \tau_j + (\rho\tau)_{ij}$$

Score on Y of the *i*th individual in the *j*th treatment = Grand Mean + Overall offset for Individual *i* + Treatment offset for level *j* + Error

One Repeated Factor Linear Model (Population Model)

$$Y_{ij} = \mu_{..} + \rho_i + \tau_j + \varepsilon_{ij}$$

Score on Y of the *i*th individual in the *j*th treatment = Grand Mean + Overall offset for Individual *i* + Treatment offset for level *j* + Error

One Repeated Factor Linear Model (Sample Model)

$$Y_{ij} = \bar{Y}_{..} + r_i + t_j + e_{ij}$$

Score on Y of the *i*th individual in the *j*th treatment = Grand Mean + Overall offset for Individual *i* + Treatment offset for level *j* + Error

Fixed vs. Random Factors

Fixed Factor: Levels of the factor are *fixed*: the levels observed in the study represent the actual levels of experimental interest

Random Factor: Levels of the factor are *random*: the levels observed in the study represent a sample from all the possible levels

	Wine 1	Wine 2	Wine 3	Wine 4	
Judge 1	Y_{11}	Y_{12}	Y_{13}	Y_{14}	$\bar{Y}_{1\cdot}$
Judge 2	Y_{21}	Y_{22}	Y_{23}	Y_{24}	$\bar{Y}_{2\cdot}$
Judge 3	Y_{31}	Y_{32}	Y_{33}	Y_{34}	$\bar{Y}_{3\cdot}$
Judge 4	Y_{41}	Y_{42}	Y_{43}	Y_{44}	$\bar{Y}_{4\cdot}$
Judge 5	Y_{51}	Y_{52}	Y_{53}	Y_{54}	$\bar{Y}_{5\cdot}$
Judge 6	Y_{61}	Y_{62}	Y_{63}	Y_{64}	$\bar{Y}_{6\cdot}$
Judge 7	Y_{71}	Y_{72}	Y_{73}	Y_{74}	$\bar{Y}_{7\cdot}$
Judge 8	Y_{81}	Y_{82}	Y_{83}	Y_{84}	$\bar{Y}_{8\cdot}$
Judge 9	Y_{91}	Y_{92}	Y_{93}	Y_{94}	$\bar{Y}_{9\cdot}$
...
	$\bar{Y}_{\cdot 1}$	$\bar{Y}_{\cdot 2}$	$\bar{Y}_{\cdot 3}$	$\bar{Y}_{\cdot 4}$	

Fixed Levels

Random Levels

Fixed vs. Random Factors

Fixed Factor: Levels of the factor are *fixed*: the levels observed in the study represent the actual levels of experimental interest

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Judge 3	Y_{31}	Y_{32}	Y_{33}	Y_{34}	$\bar{Y}_{3\cdot}$
Judge 4	Y_{41}	Y_{42}	Y_{43}	Y_{44}	$\bar{Y}_{4\cdot}$
Judge 5	Y_{51}	Y_{52}	Y_{53}	Y_{54}	$\bar{Y}_{5\cdot}$
Judge 6	Y_{61}	Y_{62}	Y_{63}	Y_{64}	$\bar{Y}_{6\cdot}$
Judge 7	Y_{71}	Y_{72}	Y_{73}	Y_{74}	$\bar{Y}_{7\cdot}$
Judge 8	Y_{81}	Y_{82}	Y_{83}	Y_{84}	$\bar{Y}_{8\cdot}$
Judge 9	Y_{91}	Y_{92}	Y_{93}	Y_{94}	$\bar{Y}_{9\cdot}$
...
	$\bar{Y}_{\cdot 1}$	$\bar{Y}_{\cdot 2}$	$\bar{Y}_{\cdot 3}$	$\bar{Y}_{\cdot 4}$	

Fixed Levels

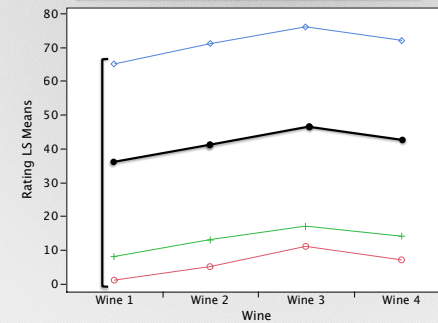
Random Levels

	Wine 1	Wine 2	Wine 3	Wine 4	
Judge 1	Y_{11}	Y_{12}	Y_{13}	Y_{14}	$\bar{Y}_{1\cdot}$
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Judge 3	Y_{31}	Y_{32}	Y_{33}	Y_{34}	$\bar{Y}_{3\cdot}$

	Wine 1	Wine 2	Wine 3	Wine 4	
Judge 1	1	5	11	7	\bar{Y}_1
Judge 2	8	13	17	14	\bar{Y}_2
Judge 3	65	71	76	72	\bar{Y}_3

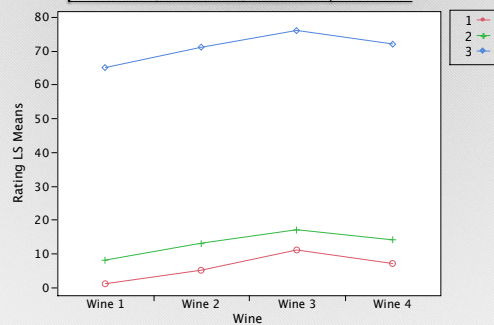
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	Wine 1	Wine 2	Wine 3	Wine 4	
Judge 1	1	5	11	7	\bar{Y}_1
Judge 2	8	13	17	14	\bar{Y}_2
Judge 3	65	71	76	72	\bar{Y}_3



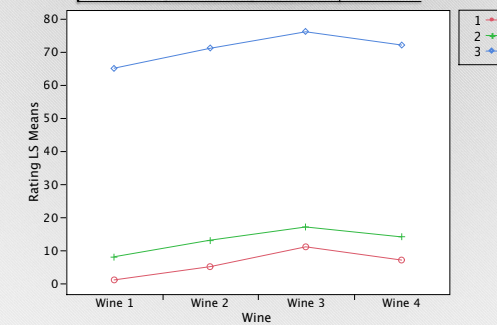
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	Wine 1	Wine 2	Wine 3	Wine 4	
Judge 1	1	5	11	7	\bar{Y}_1
Judge 2	8	13	17	14	\bar{Y}_2
Judge 3	65	71	76	72	\bar{Y}_3

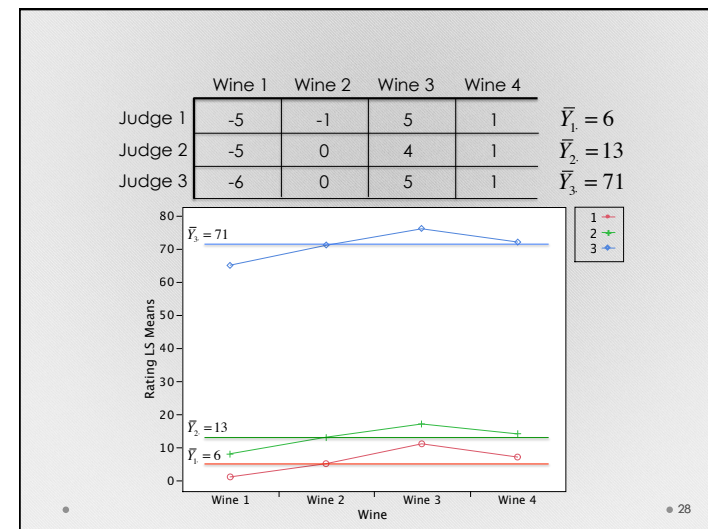
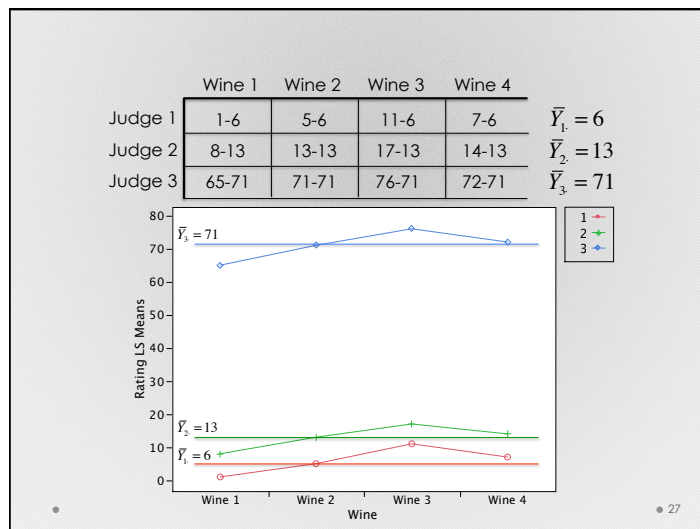
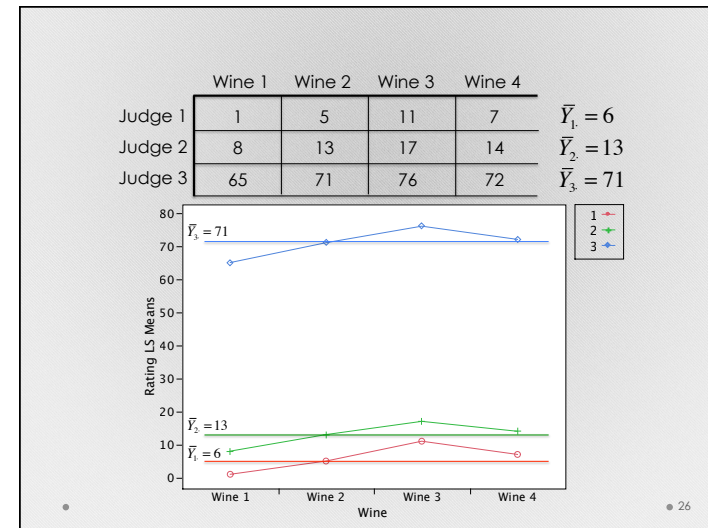
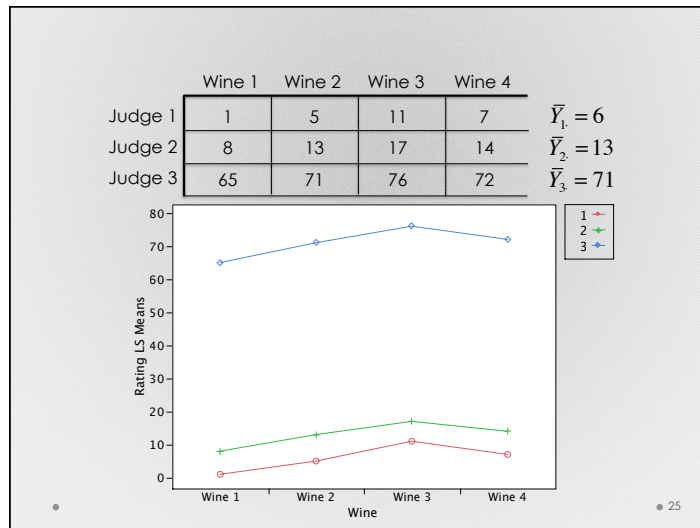


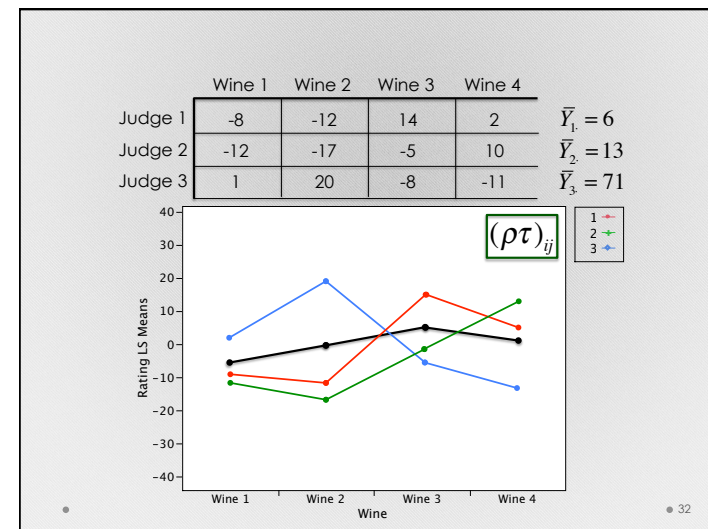
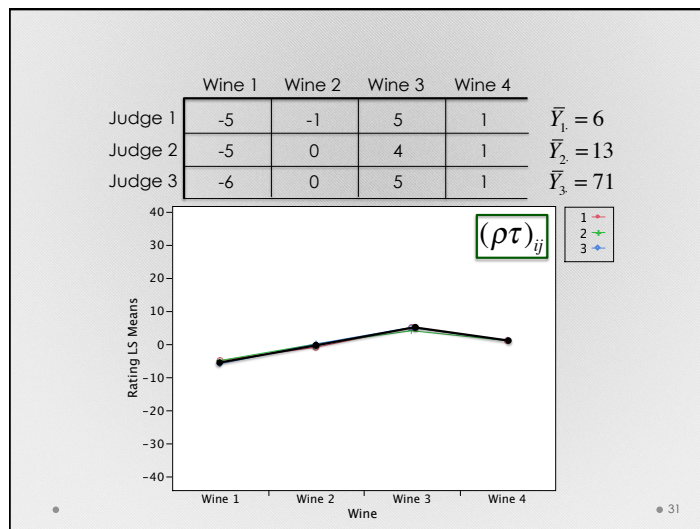
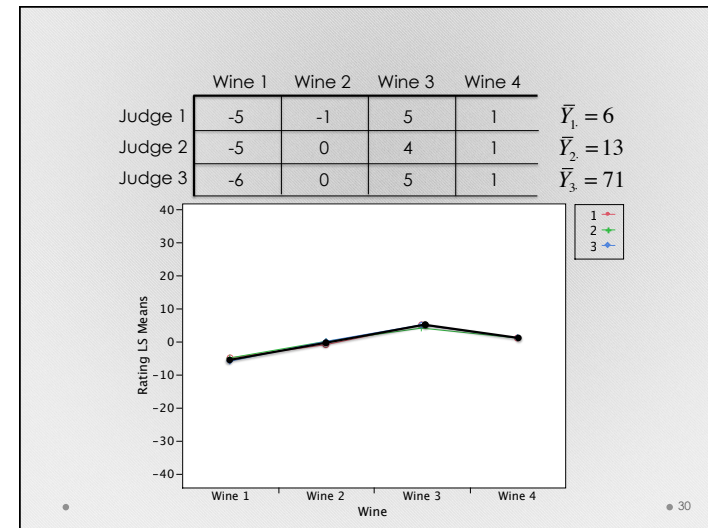
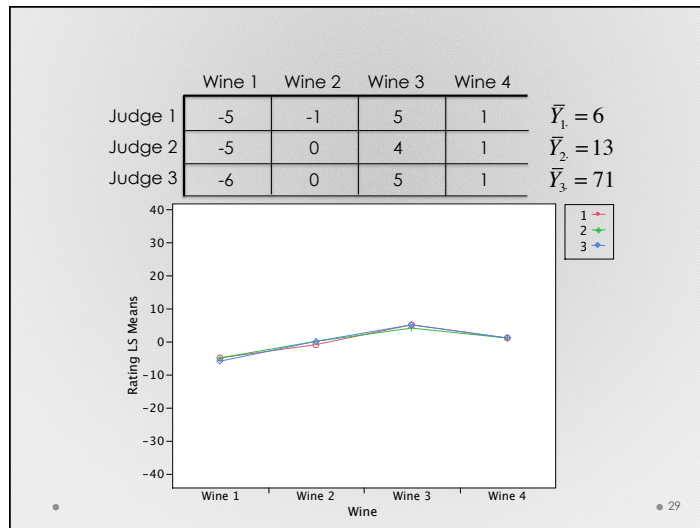
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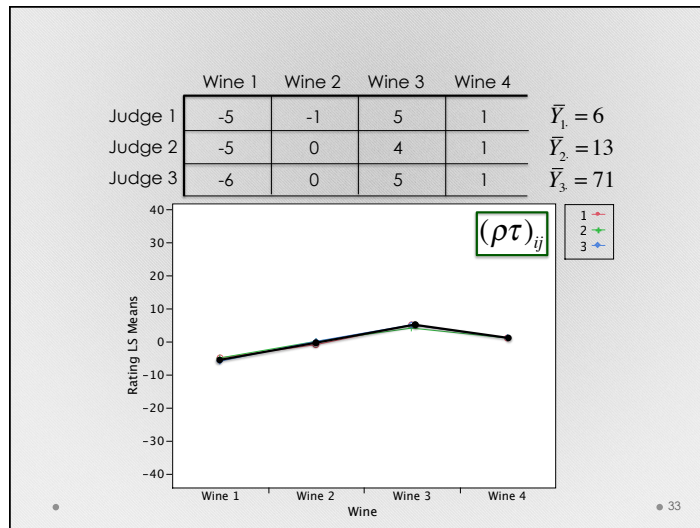
	Wine 1	Wine 2	Wine 3	Wine 4	
Judge 1	1	5	11	7	\bar{Y}_1
Judge 2	8	13	17	14	\bar{Y}_2
Judge 3	65	71	76	72	\bar{Y}_3



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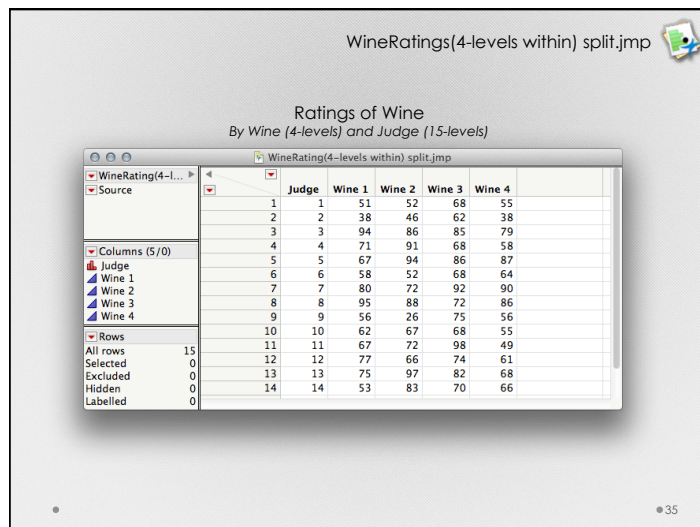


	Wine 1	Wine 2	Wine 3	Wine 4	
Judge 1	Y_{11}	Y_{12}	Y_{13}	Y_{14}	\bar{Y}_1
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Judge 5	Y_{51}	Y_{52}	Y_{53}	Y_{54}	\bar{Y}_5
Judge 6	Y_{61}	Y_{62}	Y_{63}	Y_{64}	\bar{Y}_6
Judge 7	Y_{71}	Y_{72}	Y_{73}	Y_{74}	\bar{Y}_7
Judge 8	Y_{81}	Y_{82}	Y_{83}	Y_{84}	\bar{Y}_8
Judge 9	Y_{91}	Y_{92}	Y_{93}	Y_{94}	\bar{Y}_9
...
	\bar{Y}_1	\bar{Y}_2	\bar{Y}_3	\bar{Y}_4	

Random Levels

Fixed Levels

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Data Arrangements

Split: Observations from the same unit are represented across columns

Stacked: Observations from the same unit are represented across rows

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WineRatings(4-levels within) split.jmp

Ratings of Wine
By Wine (4-levels) and Judge (15-levels)

Judge	Wine 1	Wine 2	Wine 3	Wine 4
1	1	51	52	68
2	2	38	46	62
3	3	94	86	85
4	4	71	91	68
5	5	67	94	86
6	6	58	52	68
7	7	80	72	92
8	8	95	88	72
9	9	56	26	75
10	10	62	67	68
11	11	67	72	98
12	12	77	66	74
13	13	75	97	82
14	14	53	83	70

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WineRatings(4-levels within) stacked.jmp

Ratings of Wine
By Wine (4-levels) and Judge (15-levels)

Judge	Wine	Rating
1	1 Wine 1	51
2	1 Wine 2	52
3	1 Wine 3	68
4	1 Wine 4	55
5	2 Wine 1	38
6	2 Wine 2	46
7	2 Wine 3	62
8	2 Wine 4	38
9	3 Wine 1	94
10	3 Wine 2	86
11	3 Wine 3	85
12	3 Wine 4	79
13	4 Wine 1	71
14	4 Wine 2	91

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Restructuring Data:
Stacking Repeated Observations

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WineRatings(4-levels within) stacked.jmp

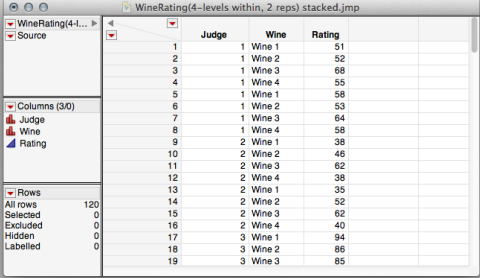
Ratings of Wine
By Wine (4-levels) and Judge (15-levels)

Judge	Wine	Rating
1	1 Wine 1	51
2	1 Wine 2	52
3	1 Wine 3	68
4	1 Wine 4	55
5	2 Wine 1	38
6	2 Wine 2	46
7	2 Wine 3	62
8	2 Wine 4	38
9	3 Wine 1	94
10	3 Wine 2	86
11	3 Wine 3	85
12	3 Wine 4	79
13	4 Wine 1	71
14	4 Wine 2	91

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WineRating(4-levels within, 2 reps) stacked.jmp

Ratings of Wine
By Wine (4-levels) and Judge (15-levels), with replication



	Judge	Wine	Rating
1	1	Wine 1	51
2	1	Wine 2	52
3	1	Wine 3	68
4	1	Wine 4	55
5	1	Wine 1	58
6	1	Wine 2	53
7	1	Wine 3	64
8	1	Wine 4	58
9	2	Wine 1	38
10	2	Wine 2	46
11	2	Wine 3	62
12	2	Wine 4	38
13	2	Wine 1	35
14	2	Wine 2	52
15	2	Wine 3	62
16	2	Wine 4	40
17	3	Wine 1	94
18	3	Wine 2	86
19	3	Wine 3	85

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