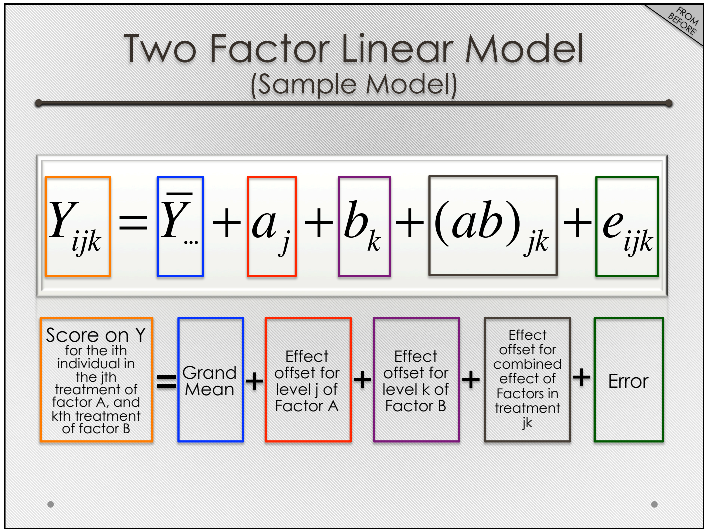
Multifactor Experimental Designs

with more than 2 factors



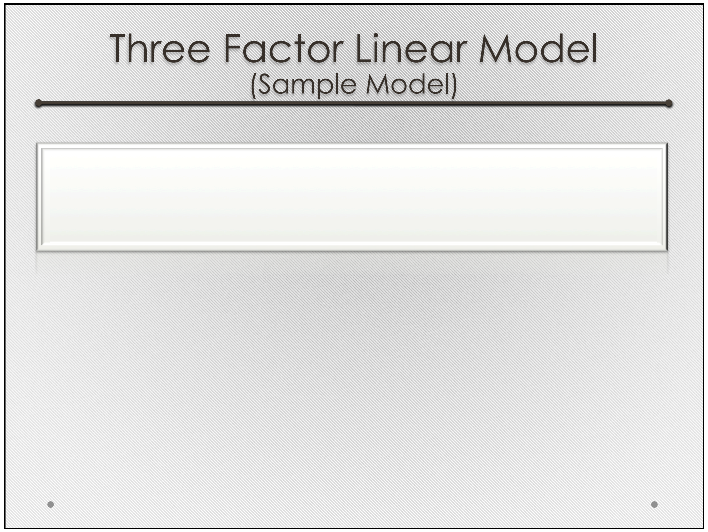
Two Factor Linear Model

(Sample Model)

FROM BEFORE

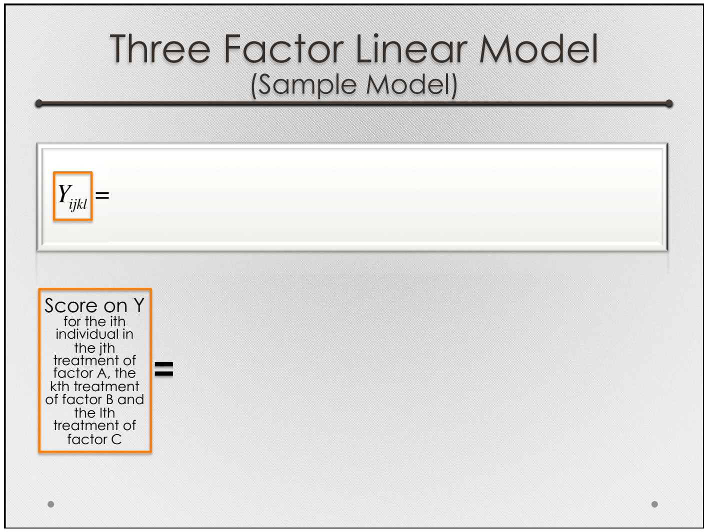
$$\boxed{Y_{ijk}} = \boxed{\bar{Y}_{...}} + \boxed{a_j} + \boxed{b_k} + \boxed{(ab)_{jk}} + \boxed{e_{ijk}}$$

Score on Y for the ith individual in the jth treatment of factor A, and kth treatment of factor B = Grand Mean + Effect offset for level j of Factor A + Effect offset for level k of Factor B + Effect offset for combined effect of Factors in treatment jk + Error



Three Factor Linear Model

(Sample Model)

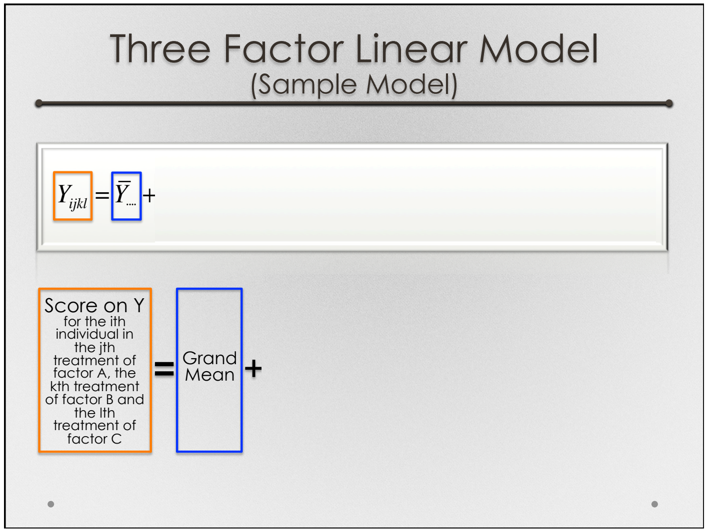


Three Factor Linear Model

(Sample Model)

$$\boxed{Y_{ijkl}} =$$

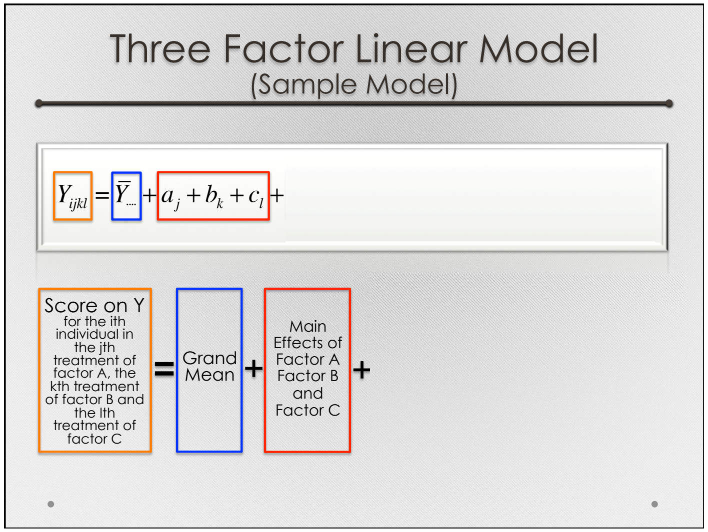
Score on Y for the ith individual in the jth treatment of factor A, the kth treatment of factor B and the lth treatment of factor C =



Three Factor Linear Model
(Sample Model)

$$\boxed{Y_{ijkl}} = \boxed{\bar{Y}_{....}} +$$

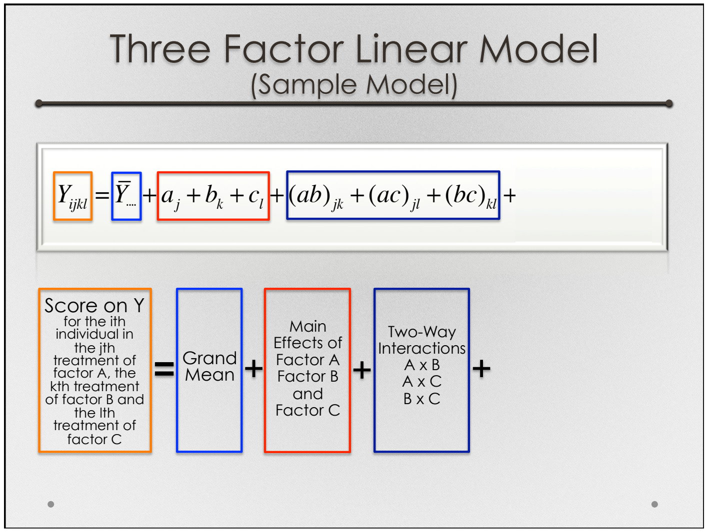
Score on Y for the ith individual in the jth treatment of factor A, the kth treatment of factor B and the lth treatment of factor C = Grand Mean +



Three Factor Linear Model
(Sample Model)

$$\boxed{Y_{ijkl}} = \boxed{\bar{Y}_{....}} + \boxed{a_j + b_k + c_l} +$$

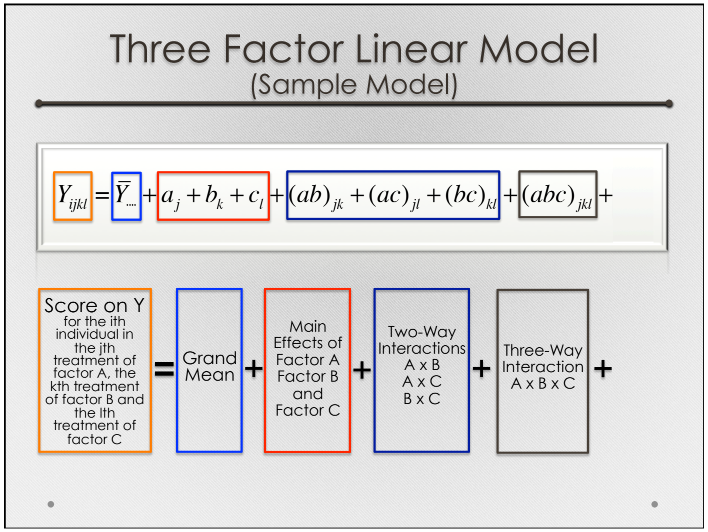
Score on Y for the ith individual in the jth treatment of factor A, the kth treatment of factor B and the lth treatment of factor C = Grand Mean + Main Effects of Factor A Factor B and Factor C +



Three Factor Linear Model
(Sample Model)

$$\boxed{Y_{ijkl}} = \boxed{\bar{Y}_{....}} + \boxed{a_j + b_k + c_l} + \boxed{(ab)_{jk} + (ac)_{jl} + (bc)_{kl}} +$$

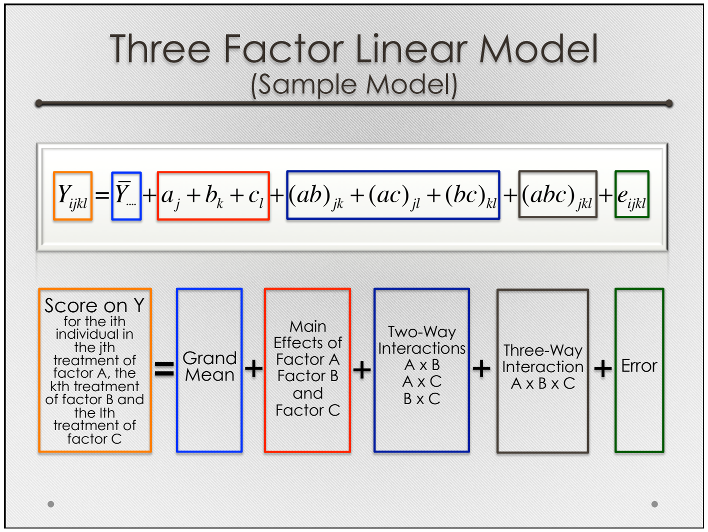
Score on Y for the ith individual in the jth treatment of factor A, the kth treatment of factor B and the lth treatment of factor C = Grand Mean + Main Effects of Factor A Factor B and Factor C + Two-Way Interactions A x B A x C B x C +



Three Factor Linear Model
(Sample Model)

$$\boxed{Y_{ijkl}} = \boxed{\bar{Y}_{....}} + \boxed{a_j + b_k + c_l} + \boxed{(ab)_{jk} + (ac)_{jl} + (bc)_{kl}} + \boxed{(abc)_{jkl}} +$$

Score on Y for the ith individual in the jth treatment of factor A, the kth treatment of factor B and the lth treatment of factor C = Grand Mean + Main Effects of Factor A Factor B and Factor C + Two-Way Interactions A x B A x C B x C + Three-Way Interaction A x B x C +

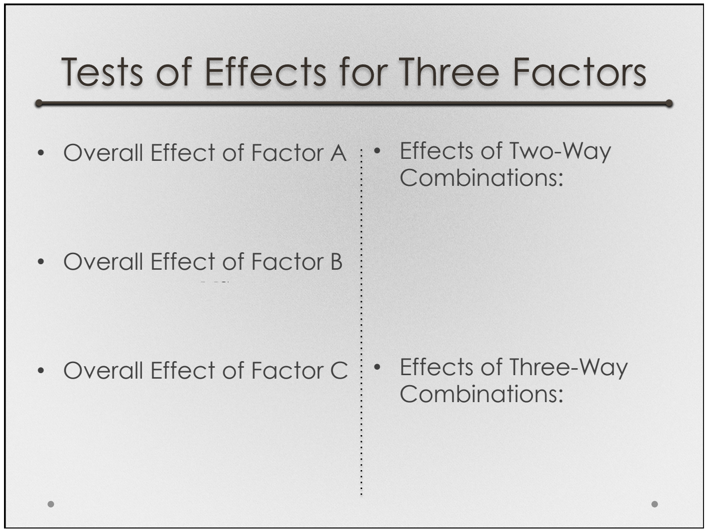


Three Factor Linear Model

(Sample Model)

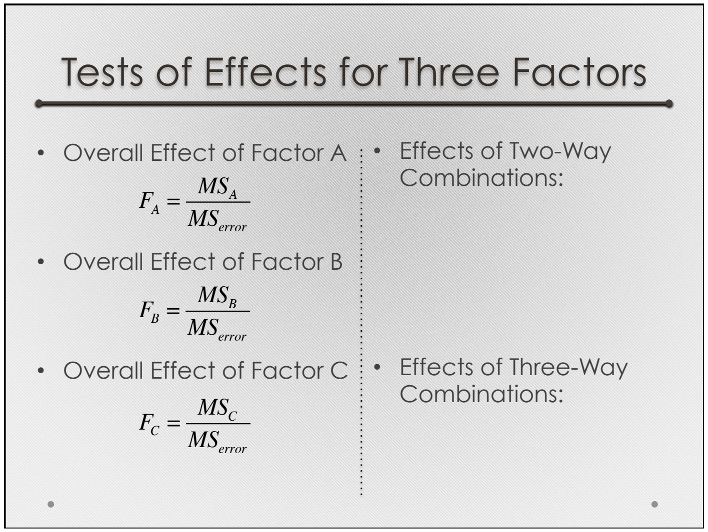
$$\boxed{Y_{ijkl}} = \boxed{\bar{Y}_{....}} + \boxed{a_j + b_k + c_l} + \boxed{(ab)_{jk} + (ac)_{jl} + (bc)_{kl}} + \boxed{(abc)_{jkl}} + \boxed{e_{ijkl}}$$

Score on Y for the ith individual in the jth treatment of factor A, the kth treatment of factor B and the lth treatment of factor C = Grand Mean + Main Effects of Factor A Factor B and Factor C + Two-Way Interactions A x B A x C B x C + Three-Way Interaction A x B x C + Error



Tests of Effects for Three Factors

- Overall Effect of Factor A
- Overall Effect of Factor B
- Overall Effect of Factor C
- Effects of Two-Way Combinations:
- Effects of Three-Way Combinations:



Tests of Effects for Three Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

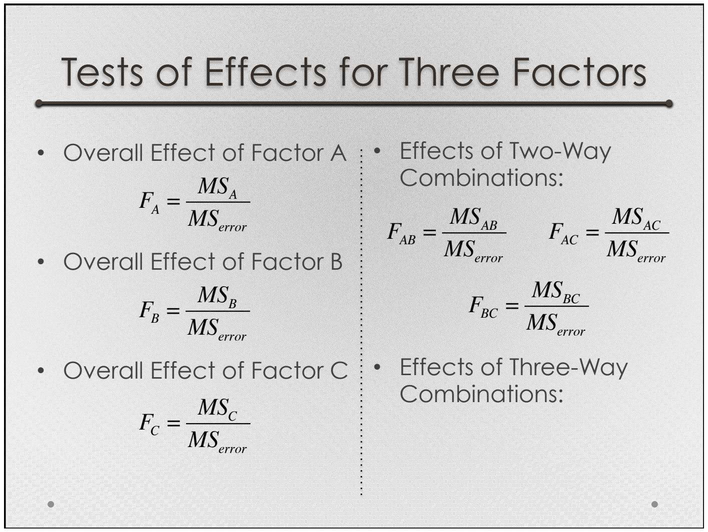
- Overall Effect of Factor B

$$F_B = \frac{MS_B}{MS_{error}}$$

- Overall Effect of Factor C

$$F_C = \frac{MS_C}{MS_{error}}$$

- Effects of Two-Way Combinations:
- Effects of Three-Way Combinations:



Tests of Effects for Three Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

- Overall Effect of Factor B

$$F_B = \frac{MS_B}{MS_{error}}$$

- Overall Effect of Factor C

$$F_C = \frac{MS_C}{MS_{error}}$$

- Effects of Two-Way Combinations:

$$F_{AB} = \frac{MS_{AB}}{MS_{error}} \quad F_{AC} = \frac{MS_{AC}}{MS_{error}}$$

$$F_{BC} = \frac{MS_{BC}}{MS_{error}}$$

- Effects of Three-Way Combinations:

Tests of Effects for Three Factors

- Overall Effect of Factor A

$$F_A = \frac{MS_A}{MS_{error}}$$

- Overall Effect of Factor B

$$F_B = \frac{MS_B}{MS_{error}}$$

- Overall Effect of Factor C

$$F_C = \frac{MS_C}{MS_{error}}$$

- Effects of Two-Way Combinations:

$$F_{AB} = \frac{MS_{AB}}{MS_{error}} \qquad F_{AC} = \frac{MS_{AC}}{MS_{error}}$$

$$F_{BC} = \frac{MS_{BC}}{MS_{error}}$$

- Effects of Three-Way Combinations:

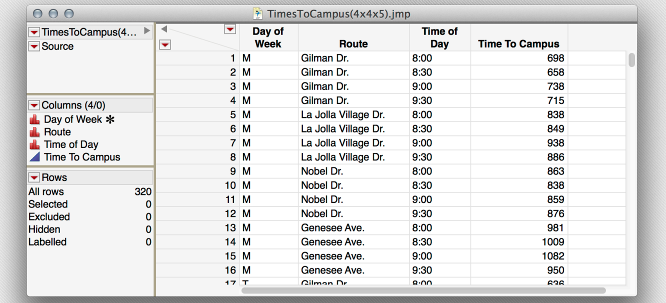
$$F_{ABC} = \frac{MS_{ABC}}{MS_{error}}$$

TimesToCampus(4x4x5)

(IBRD)*

Time (in seconds) to Campus

By Route (4-levels), Time of Morning (4-levels), and Day of Week (5-levels)



Columns (4/0)
- Day of Week ✻
- Route
- Time of Day
- Time To Campus

Rows

All rows	320
Selected	0
Excluded	0
Hidden	0
Labelled	0

	Day of Week	Route	Time of Day	Time To Campus
1	M	Gilman Dr.	8:00	698
2	M	Gilman Dr.	8:30	658
3	M	Gilman Dr.	9:00	738
4	M	Gilman Dr.	9:30	715
5	M	La Jolla Village Dr.	8:00	838
6	M	La Jolla Village Dr.	8:30	849
7	M	La Jolla Village Dr.	9:00	938
8	M	La Jolla Village Dr.	9:30	886
9	M	Nobel Dr.	8:00	863
10	M	Nobel Dr.	8:30	838
11	M	Nobel Dr.	9:00	859
12	M	Nobel Dr.	9:30	876
13	M	Genesee Ave.	8:00	981
14	M	Genesee Ave.	8:30	1009
15	M	Genesee Ave.	9:00	1082
16	M	Genesee Ave.	9:30	950

Multifactor Experimental Designs

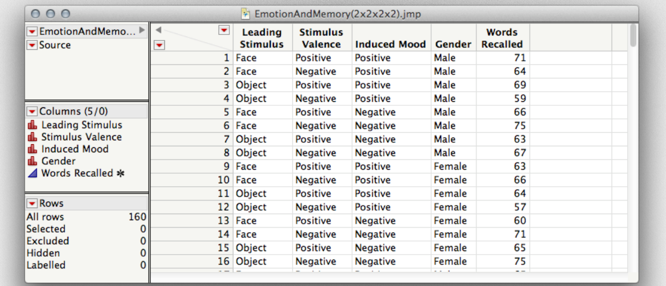
with 4 factors

EmotionAndMemory(2x2x2x2).jmp

Emotion and Memory

By Stimulus Type (2-levels), Valence (2-levels), Induced Mood (2-levels) and Gender (2-levels)

Total Words Recalled, 10 blocks of 10 words.

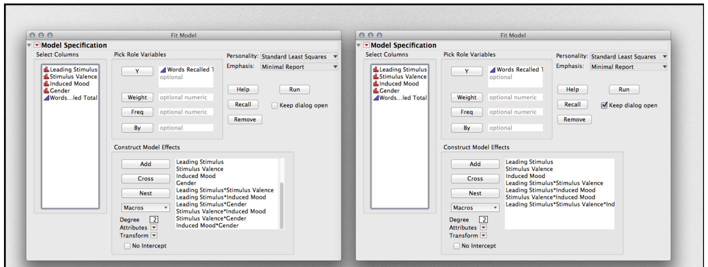


Columns (5/0)
- Leading Stimulus
- Stimulus Valence
- Induced Mood
- Gender
- Words Recalled ✻

Rows

All rows	160
Selected	0
Excluded	0
Hidden	0
Labelled	0

	Leading Stimulus	Stimulus Valence	Induced Mood	Gender	Words Recalled
1	Face	Positive	Positive	Male	71
2	Face	Negative	Positive	Male	64
3	Object	Positive	Positive	Male	69
4	Object	Negative	Positive	Male	59
5	Face	Positive	Negative	Male	66
6	Face	Negative	Negative	Male	75
7	Object	Positive	Negative	Male	63
8	Object	Negative	Negative	Male	67
9	Face	Positive	Positive	Female	63
10	Face	Negative	Positive	Female	66
11	Object	Positive	Positive	Female	64
12	Object	Negative	Positive	Female	57
13	Face	Positive	Negative	Female	60
14	Face	Negative	Negative	Female	71
15	Object	Positive	Negative	Female	65
16	Object	Negative	Negative	Female	75



Fit Model

Model Specification

Select Columns: Leading Stimulus, Stimulus Valence, Induced Mood, Gender, Words…led Total

Y: Words Recalled T

Personality: Standard Least Squares

Emphasis: Minimal Report

☐ Keep dialog open

Construct Model Effects:

- Leading Stimulus
- Stimulus Valence
- Induced Mood
- Gender
- Leading Stimulus*Stimulus Valence
- Leading Stimulus*Induced Mood
- Leading Stimulus*Gender
- Stimulus Valence*Induced Mood
- Stimulus Valence*Gender
- Induced Mood*Gender

Degree: 2

☐ No Intercept

Fit Model

Model Specification

Select Columns: Leading Stimulus, Stimulus Valence, Induced Mood, Gender, Words…led Total

Y: Words Recalled T

Personality: Standard Least Squares

Emphasis: Minimal Report

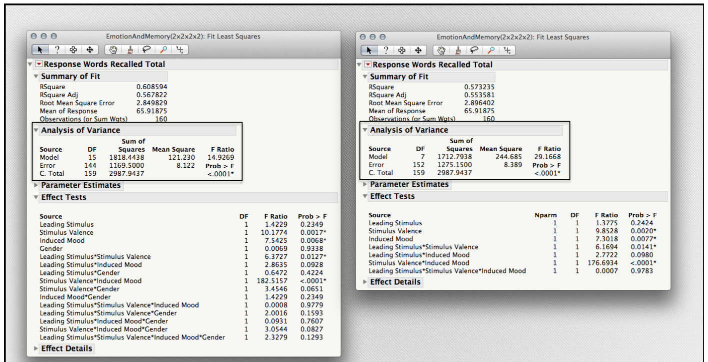
☑ Keep dialog open

Construct Model Effects:

- Leading Stimulus
- Stimulus Valence
- Induced Mood
- Leading Stimulus*Stimulus Valence
- Leading Stimulus*Induced Mood
- Stimulus Valence*Induced Mood
- Leading Stimulus*Stimulus Valence*Ind

Degree: 2

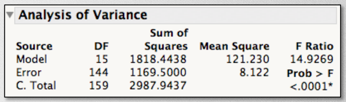
☐ No Intercept



```
EmotionAndMemory(2x2x2x2): Fit Least Squares
Response Words Recalled Total
 Summary of Fit
 RSquare                        0.608594
 RSquare Adj                    0.567822
 Root Mean Square Error         2.849829
 Mean of Response               65.91875
 Observations (or Sum Wgts)          160
 Analysis of Variance
                      Sum of
 Source       DF     Squares  Mean Square    F Ratio
 Model        15   1818.4438      121.230    14.9269
 Error       144   1169.5000        8.122   Prob > F
 C. Total    159   2987.9437                <.0001*
 Parameter Estimates
 Effect Tests
 Source                                                   DF    F Ratio   Prob > F
 Leading Stimulus                                          1     1.4229    0.2349
 Stimulus Valence                                          1    10.1774    0.0017*
 Induced Mood                                              1     7.5425    0.0068*
 Gender                                                    1     0.0069    0.9338
 Leading Stimulus*Stimulus Valence                         1     6.3727    0.0127*
 Leading Stimulus*Induced Mood                             1     2.8635    0.0928
 Leading Stimulus*Gender                                   1     0.6472    0.4224
 Stimulus Valence*Induced Mood                             1   182.5157    <.0001*
 Stimulus Valence*Gender                                   1     3.4546    0.0651
 Induced Mood*Gender                                       1     1.4229    0.2349
 Leading Stimulus*Stimulus Valence*Induced Mood            1     0.0008    0.9779
 Leading Stimulus*Stimulus Valence*Gender                  1     2.0016    0.1593
 Leading Stimulus*Induced Mood*Gender                      1     0.0931    0.7607
 Stimulus Valence*Induced Mood*Gender                      1     3.0544    0.0827
 Leading Stimulus*Stimulus Valence*Induced Mood*Gender     1     2.3279    0.1293
 Effect Details
```

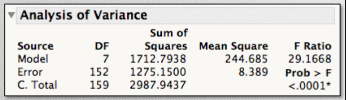
```
EmotionAndMemory(2x2x2x2): Fit Least Squares
Response Words Recalled Total
 Summary of Fit
 RSquare                        0.573235
 RSquare Adj                    0.553581
 Root Mean Square Error         2.896402
 Mean of Response               65.91875
 Observations (or Sum Wgts)          160
 Analysis of Variance
                      Sum of
 Source       DF     Squares  Mean Square    F Ratio
 Model         7   1712.7938      244.685    29.1668
 Error       152   1275.1500        8.389   Prob > F
 C. Total    159   2987.9437                <.0001*
 Parameter Estimates
 Effect Tests
 Source                                            Nparm   DF    F Ratio   Prob > F
 Leading Stimulus                                      1    1     1.3775    0.2424
 Stimulus Valence                                      1    1     9.8528    0.0020*
 Induced Mood                                          1    1     7.3018    0.0077*
 Leading Stimulus*Stimulus Valence                     1    1     6.1694    0.0141*
 Leading Stimulus*Induced Mood                         1    1     2.7722    0.0980
 Stimulus Valence*Induced Mood                         1    1   176.6934    <.0001*
 Leading Stimulus*Stimulus Valence*Induced Mood        1    1     0.0007    0.9783
 Effect Details
```

Full Model



```
Analysis of Variance
                      Sum of
 Source       DF     Squares  Mean Square    F Ratio
 Model        15   1818.4438      121.230    14.9269
 Error       144   1169.5000        8.122   Prob > F
 C. Total    159   2987.9437                <.0001*
```

Reduced Model



```
Analysis of Variance
                      Sum of
 Source       DF     Squares  Mean Square    F Ratio
 Model         7   1712.7938      244.685    29.1668
 Error       152   1275.1500        8.389   Prob > F
 C. Total    159   2987.9437                <.0001*
```

FROM BEFORE

The General Linear Test

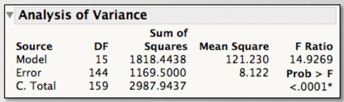
$$F = \frac{\left(\dfrac{\text{Reduction in Error}}{\#\ \text{Added Parameters}}\right)}{\textit{Baseline}\ \text{Error}}$$

FROM BEFORE

The General Linear Test

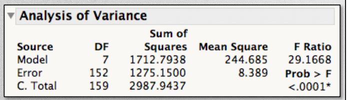
$$F = \frac{\dfrac{SS_{error}(R) - SS_{error}(F)}{df_{error}(R) - df_{error}(F)}}{\dfrac{SS_{error}(F)}{df_{error}(F)}}$$

Full Model



```
Analysis of Variance
                          Sum of
Source       DF          Squares   Mean Square    F Ratio
Model        15        1818.4438       121.230    14.9269
Error       144        1169.5000         8.122   Prob > F
C. Total    159        2987.9437                  <.0001*
```

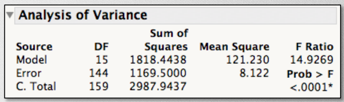
Reduced Model



```
Analysis of Variance
                          Sum of
Source       DF          Squares   Mean Square    F Ratio
Model         7        1712.7938       244.685    29.1668
Error       152        1275.1500         8.389   Prob > F
C. Total    159        2987.9437                  <.0001*
```

$$F = \frac{\dfrac{SS_{error}(R) - SS_{error}(F)}{df_{error}(R) - df_{error}(F)}}{\dfrac{SS_{error}(F)}{df_{error}(F)}}$$

Full Model



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Analysis of Variance
                          Sum of
Source       DF          Squares   Mean Square    F Ratio
Model        15        1818.4438       121.230    14.9269
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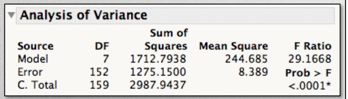
```

Reduced Model



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Analysis of Variance
                          Sum of
Source       DF          Squares   Mean Square    F Ratio
Model         7        1712.7938       244.685    29.1668
Error       152        1275.1500         8.389   Prob > F
C. Total    159        2987.9437                  <.0001*
```
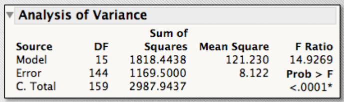

$$F = \frac{\dfrac{SS_{error}(R) - SS_{error}(F)}{df_{error}(R) - df_{error}(F)}}{MS_{error}(F)}$$

Full Model



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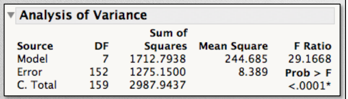
Analysis of Variance
                          Sum of
Source       DF          Squares   Mean Square    F Ratio
Model        15        1818.4438       121.230    14.9269
Error       144        1169.5000         8.122   Prob > F
C. Total    159        2987.9437                  <.0001*
```

Reduced Model



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Analysis of Variance
                          Sum of
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Model         7        1712.7938       244.685    29.1668
Error       152        1275.1500         8.389   Prob > F
C. Total    159        2987.9437                  <.0001*
```
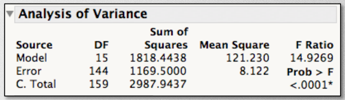

$$F = \frac{\dfrac{SS_{error}(R) - SS_{error}(F)}{df_{error}(R) - df_{error}(F)}}{MS_{error}(F)}$$

$$F = \frac{\dfrac{\qquad\qquad}{\qquad\qquad}}{\qquad\qquad} =$$

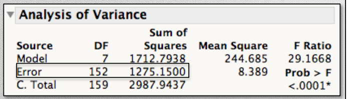
Full Model



Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	15	1818.4438	121.230	14.9269
Error	144	1169.5000	8.122	Prob > F
C. Total	159	2987.9437		<.0001*

Reduced Model



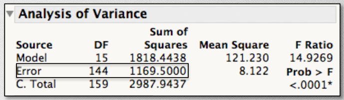
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	7	1712.7938	244.685	29.1668
Error	152	1275.1500	8.389	Prob > F
C. Total	159	2987.9437		<.0001*

$$F=\frac{\dfrac{SS_{error}(R)-SS_{error}(F)}{df_{error}(R)-df_{error}(F)}}{MS_{error}(F)}$$

$$F=\frac{\dfrac{1275.15-}{}}{}=$$

Full Model



Analysis of Variance

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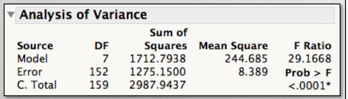
Model	15	1818.4438	121.230	14.9269
Error	144	1169.5000	8.122	Prob > F
C. Total	159	2987.9437		<.0001*

Reduced Model



Analysis of Variance

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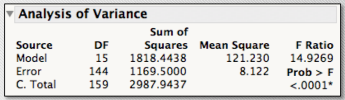
Model	7	1712.7938	244.685	29.1668
Error	152	1275.1500	8.389	Prob > F
C. Total	159	2987.9437		<.0001*

$$F=\frac{\dfrac{SS_{error}(R)-SS_{error}(F)}{df_{error}(R)-df_{error}(F)}}{MS_{error}(F)}$$

$$F=\frac{\dfrac{1275.15-1169.5}{}}{}=$$

Full Model



Analysis of Variance

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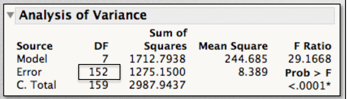

Analysis of Variance

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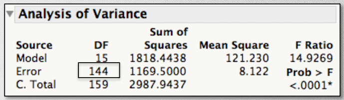
Model	7	1712.7938	244.685	29.1668
Error	152	1275.1500	8.389	Prob > F
C. Total	159	2987.9437		<.0001*

$$F=\frac{\dfrac{SS_{error}(R)-SS_{error}(F)}{df_{error}(R)-df_{error}(F)}}{MS_{error}(F)}$$

$$F=\frac{\dfrac{1275.15-1169.5}{152-}}{}=$$

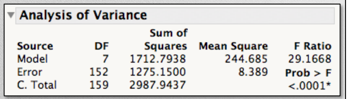
Full Model



Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	15	1818.4438	121.230	14.9269
Error	144	1169.5000	8.122	Prob > F
C. Total	159	2987.9437		<.0001*

Reduced Model



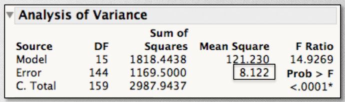
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	7	1712.7938	244.685	29.1668
Error	152	1275.1500	8.389	Prob > F
C. Total	159	2987.9437		<.0001*

$$F=\frac{\dfrac{SS_{error}(R)-SS_{error}(F)}{df_{error}(R)-df_{error}(F)}}{MS_{error}(F)}$$

$$F=\frac{\dfrac{1275.15-1169.5}{152-144}}{}=$$

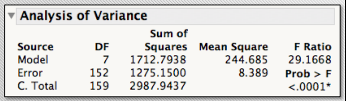
Full Model



Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	15	1818.4438	121.230	14.9269
Error	144	1169.5000	$\boxed{8.122}$	**Prob > F**
C. Total	159	2987.9437		<.0001*

Reduced Model



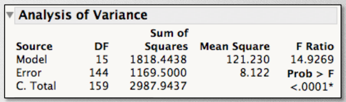
Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	7	1712.7938	244.685	29.1668
Error	152	1275.1500	8.389	**Prob > F**
C. Total	159	2987.9437		<.0001*

$$F = \frac{\dfrac{SS_{error}(R) - SS_{error}(F)}{df_{error}(R) - df_{error}(F)}}{MS_{error}(F)}$$

$$F = \frac{\dfrac{1275.15 - 1169.5}{152 - 144}}{8.122} =$$

Full Model



Analysis of Variance

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Model	15	1818.4438	121.230	14.9269
Error	144	1169.5000	8.122	**Prob > F**
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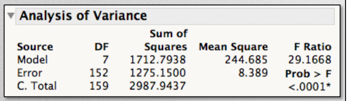

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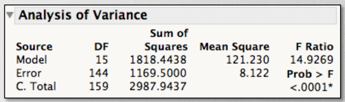
$$F = \frac{\dfrac{1275.15 - 1169.5}{152 - 144}}{8.122} =$$

Full Model



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Model	15	1818.4438	121.230	14.9269
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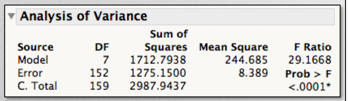
Reduced Model



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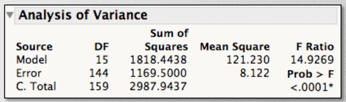
$$F = \frac{\dfrac{1275.15 - 1169.5}{152 - 144}}{8.122} = \frac{\dfrac{105.65}{8}}{8.122} =$$

Full Model



Analysis of Variance

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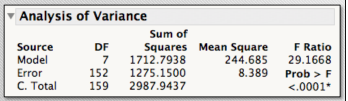
Reduced Model



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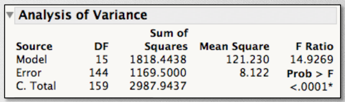
$$F = \frac{\dfrac{1275.15 - 1169.5}{152 - 144}}{8.122} = \frac{\dfrac{105.65}{8}}{8.122} = \frac{13.206}{8.122} =$$

Full Model



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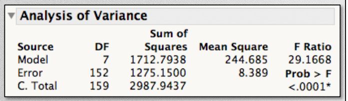
Reduced Model



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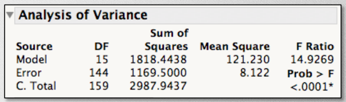
$$F = \frac{\frac{1275.15 - 1169.5}{152 - 144}}{8.122} = \frac{\frac{105.65}{8}}{8.122} = \frac{13.206}{8.122} = 1.626$$

Full Model



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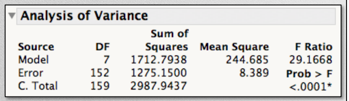
Reduced Model



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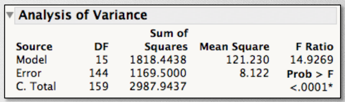
$$df_{numerator} = 8$$

Full Model



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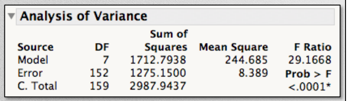

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$$df_{numerator} = 8$$

$$df_{numerator} = 144$$