Single Group Analyses	Single Group Experiments
T-test Z-test	Single Sample Design
T-test	Pre-Post Designs
Mixed-effects models	Repeated Measures Designs
Multi-Group Analyses	**Multi-Group Experiments**
T-test	Two Group Design
Analysis of Variance	Multi-Group Design
Regression Analysis	Continuous Predictor Design
Mixed Analyses	**Mixed Experiments**
Mixed-effects models	Group Design with Repeated Measurements
Analysis of Covariance	Group Design also measured on a continuous variable

1

Single Group Analyses	Single Group Experiments
T-test Z-test	**Single Sample Design**
T-test	**Pre-Post Designs**
Mixed-effects models	Repeated Measures Designs
Multi-Group Analyses	**Multi-Group Experiments**
T-test	**Two Group Design**
Analysis of Variance	Multi-Group Design
Regression Analysis	Continuous Predictor Design
Mixed Analyses	**Mixed Experiments**
Mixed-effects models	Group Design with Repeated Measurements
Analysis of Covariance	Group Design also measured on a continuous variable

2

Single Group Analyses	Single Group Experiments
T-test Z-test	Single Sample Design
T-test	Pre-Post Designs
Mixed-effects models	**Repeated Measures Designs**
Multi-Group Analyses	**Multi-Group Experiments**
T-test	Two Group Design
Analysis of Variance	**Multi-Group Design**
Regression Analysis	**Continuous Predictor Design**
Mixed Analyses	**Mixed Experiments**
Mixed-effects models	**Group Design with Repeated Measurements**
Analysis of Covariance	**Group Design also measured on a continuous variable**

3

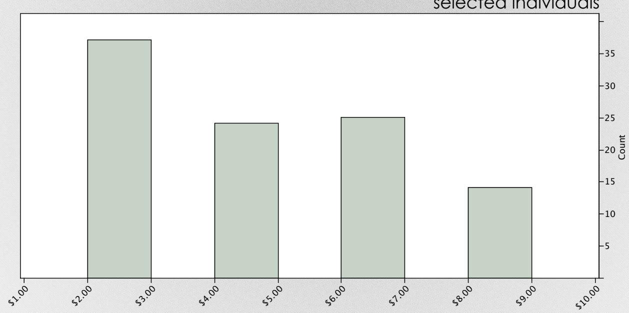
Mathematical Models

- Extend our descriptions of nature to situations with more than two groups
- Are intended to be parsimonious descriptions of effects and relationships (no more complicated than necessary)
- Can describe ***functional*** and also ***statistical*** relationships between variables

Modeling Functional Relationships

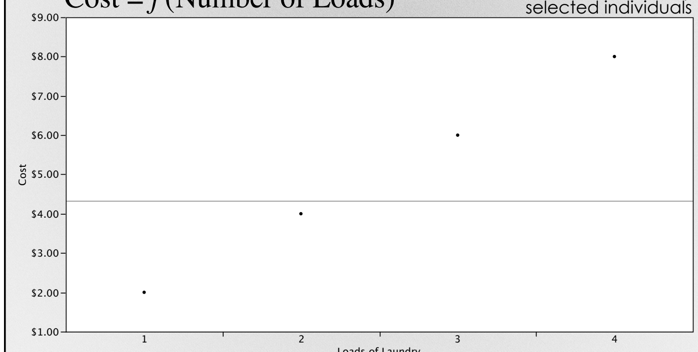
Price of Doing Laundry

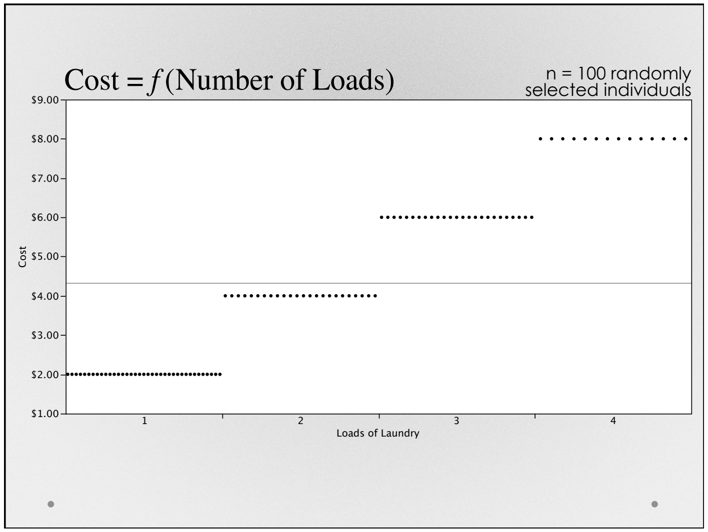
n = 100 randomly selected individuals



$\text{Cost} = f(\text{Number of Loads})$

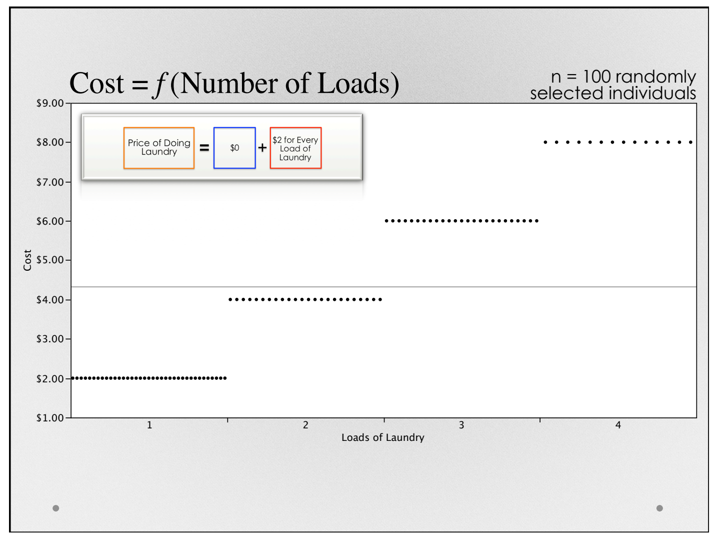
n = 100 randomly selected individuals





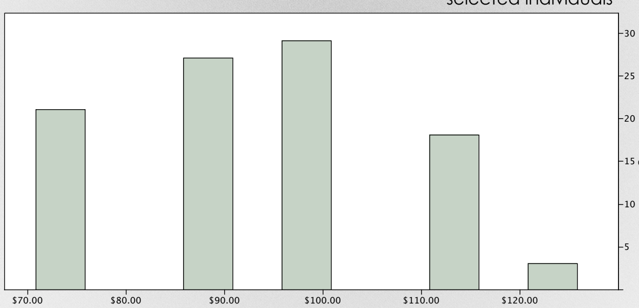
Individual's Score on Outcome Measure = Overall Constant + Effect for Some Measured Variable

Price of Doing Laundry = $0 + $2 for Every Load of Laundry



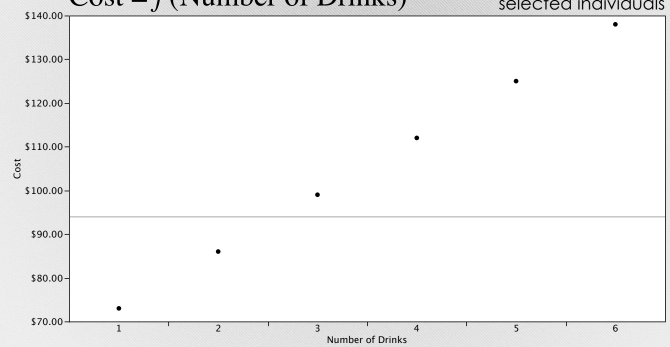
Halloween at a downtown club

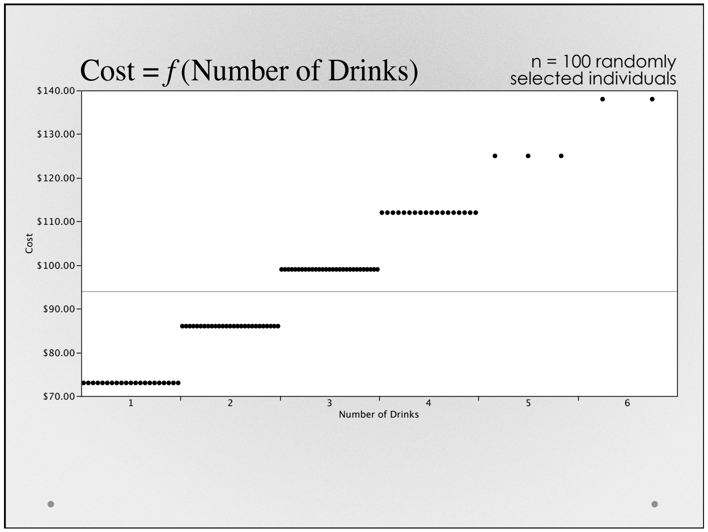
n = 100 randomly selected individuals

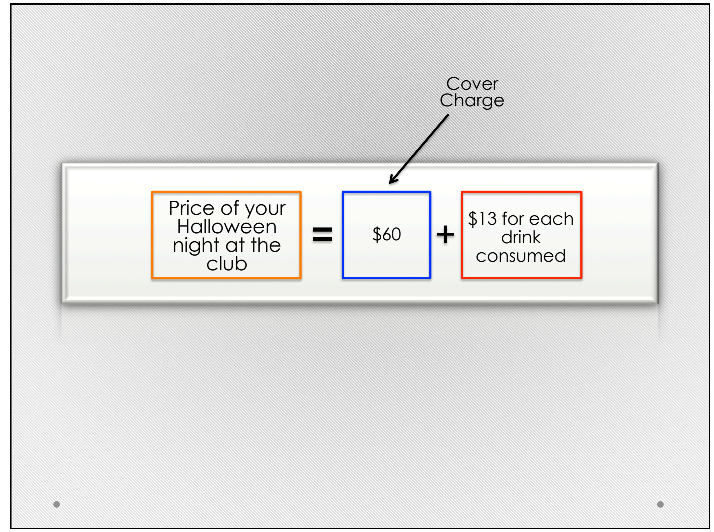


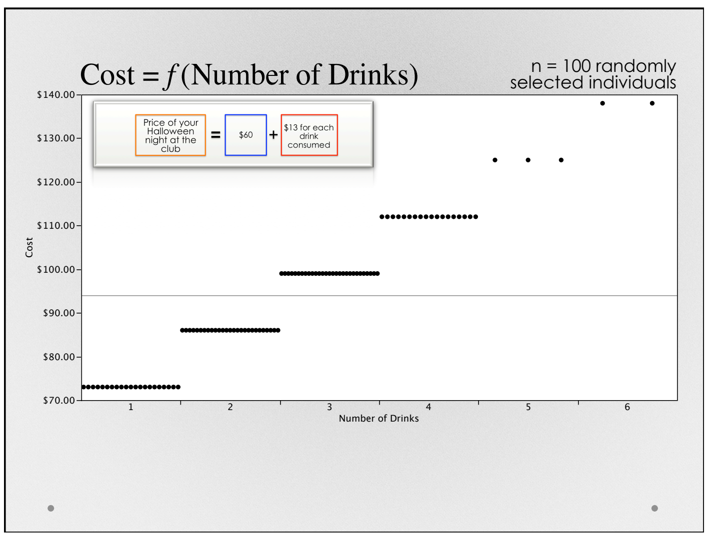
Cost = *f* (Number of Drinks)

n = 100 randomly selected individuals

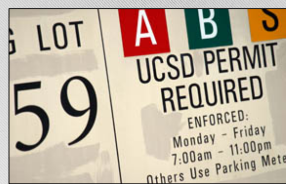



Cost = f (Number of Drinks)
n = 100 randomly selected individuals
$140.00
$130.00
$120.00
$110.00
$100.00
$90.00
$80.00
$70.00
Cost
1
2
3
4
5
6
Number of Drinks

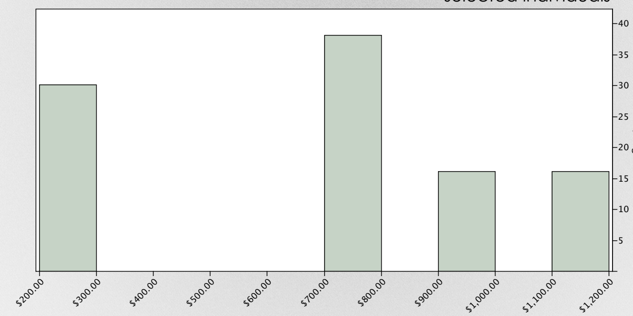

Cover Charge
Price of your Halloween night at the club
=
$60
+
$13 for each drink consumed


Cost = f (Number of Drinks)
n = 100 randomly selected individuals
Price of your Halloween night at the club
=
$60
+
$13 for each drink consumed
$140.00
$130.00
$120.00
$110.00
$100.00
$90.00
$80.00
$70.00
Cost
1
2
3
4
5
6
Number of Drinks

Parking Permit Cost

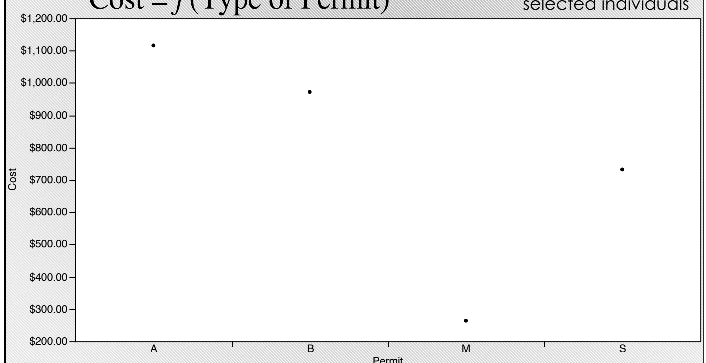


n = 100 randomly selected individuals



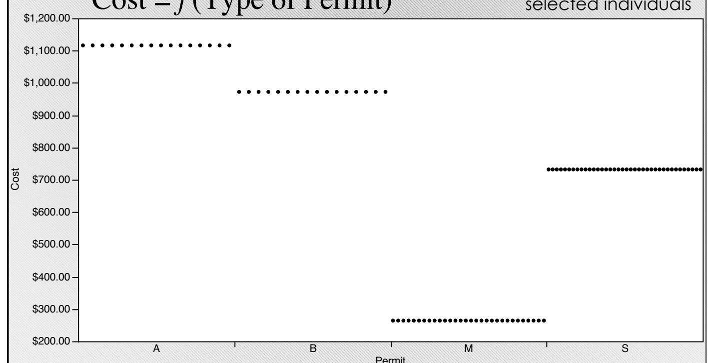
Cost = f(Type of Permit)

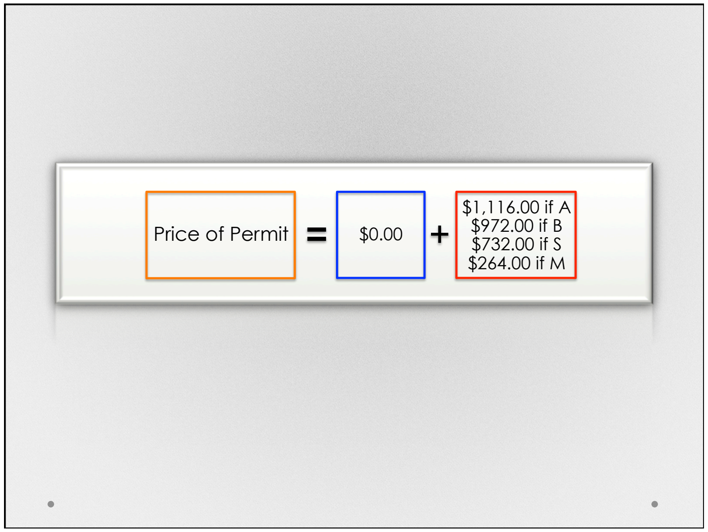
n = 100 randomly selected individuals

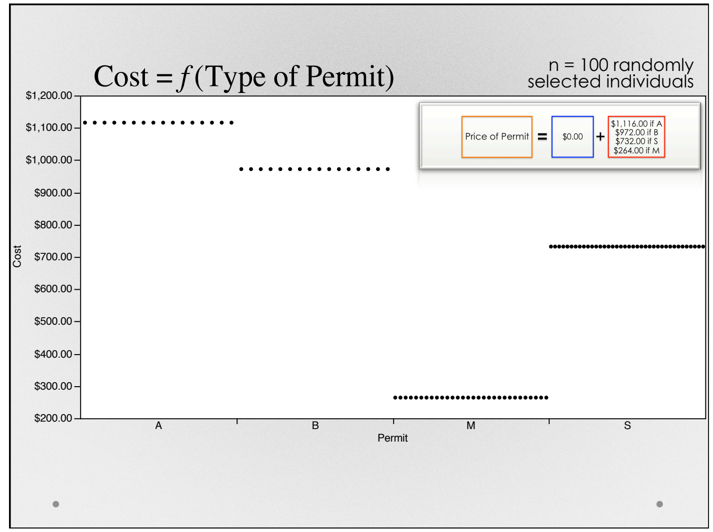


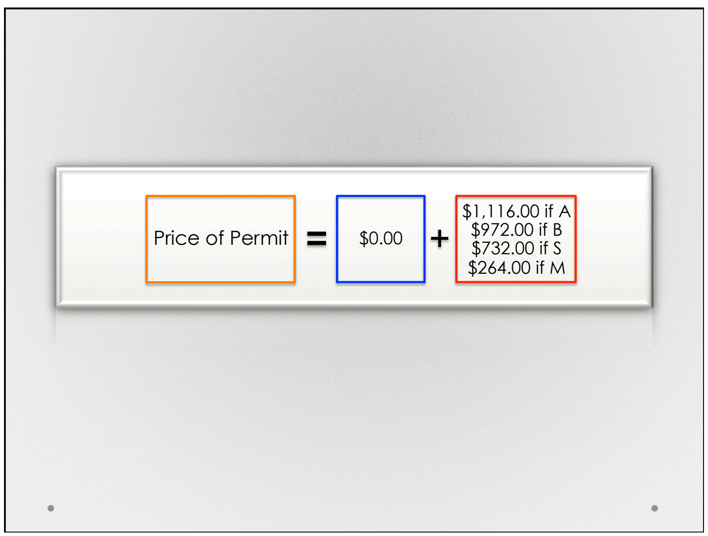
Cost = f(Type of Permit)

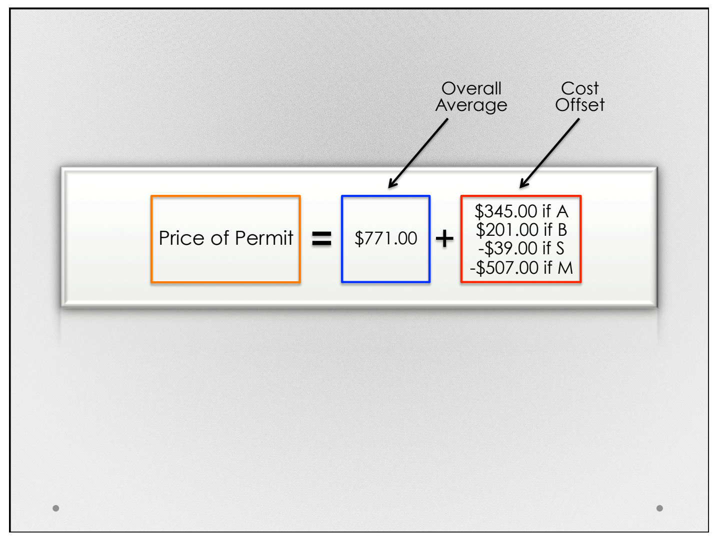
n = 100 randomly selected individuals

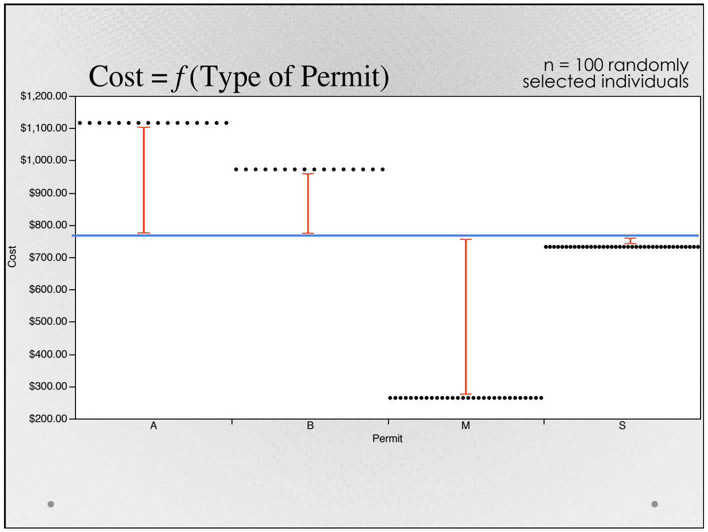


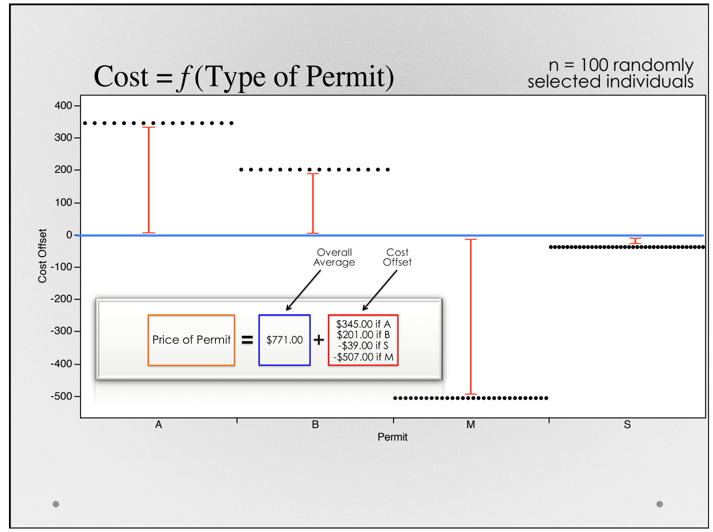

Price of Permit
=
$0.00
+
$1,116.00 if A
$972.00 if B
$732.00 if S
$264.00 if M


Cost = f (Type of Permit)
n = 100 randomly
selected individuals
Price of Permit
=
$0.00
+
$1,116.00 if A
$972.00 if B
$732.00 if S
$264.00 if M
$1,200.00
$1,100.00
$1,000.00
$900.00
$800.00
$700.00
$600.00
$500.00
$400.00
$300.00
$200.00
Cost
A
B
M
S
Permit


Price of Permit
=
$0.00
+
$1,116.00 if A
$972.00 if B
$732.00 if S
$264.00 if M


Overall
Average
Cost
Offset
Price of Permit
=
$771.00
+
$345.00 if A
$201.00 if B
-$39.00 if S
-$507.00 if M





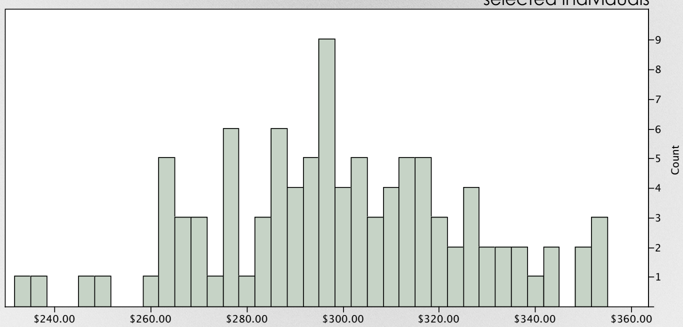
Modeling Statistical Relationships

Individual's Score on Outcome Measure = Overall Constant + Effect for Some Measured Variable

Individual's Score on Outcome Measure = Overall Constant + Effect for Some Measured Variable + Individual Error

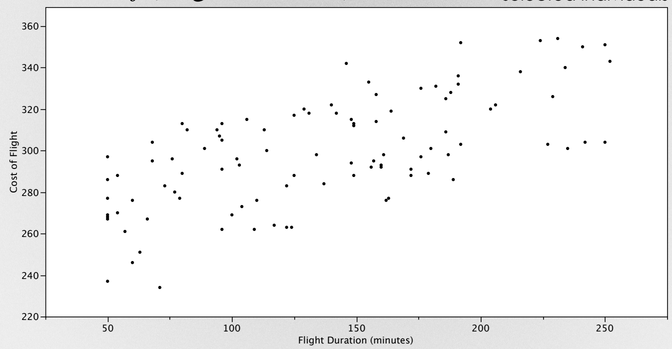
Flight Costs

n = 100 randomly selected individuals



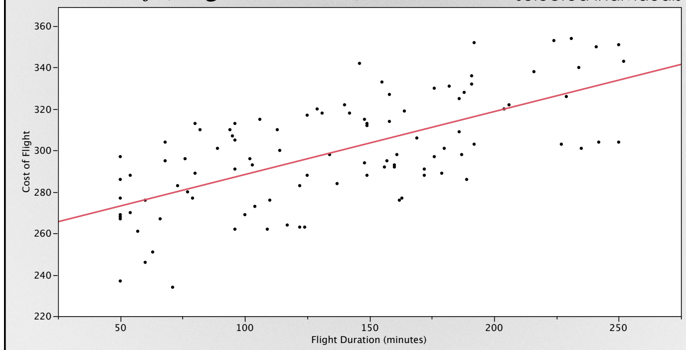
Cost = f(Flight Duration)

n = 100 randomly selected individuals



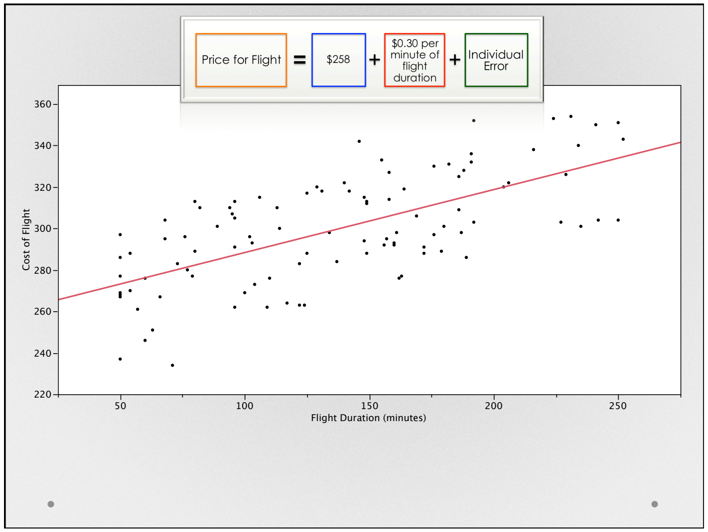
Cost = f(Flight Duration)

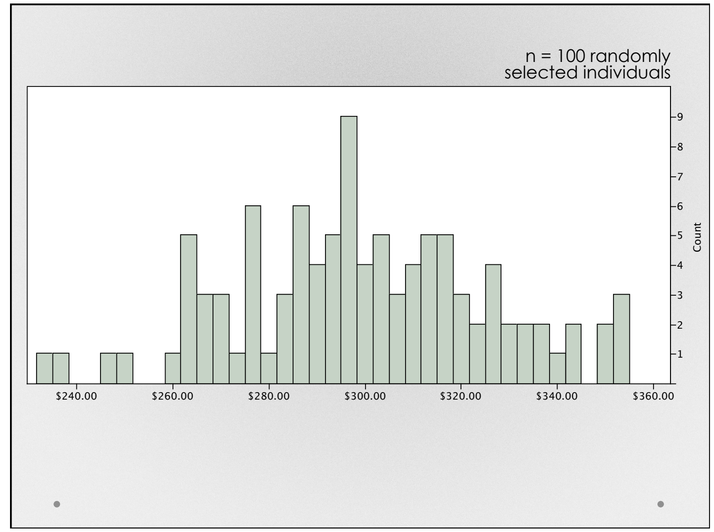
n = 100 randomly selected individuals

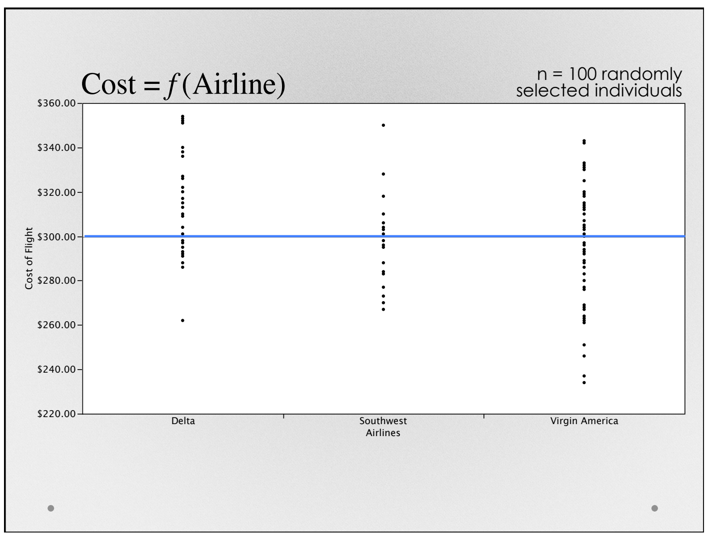


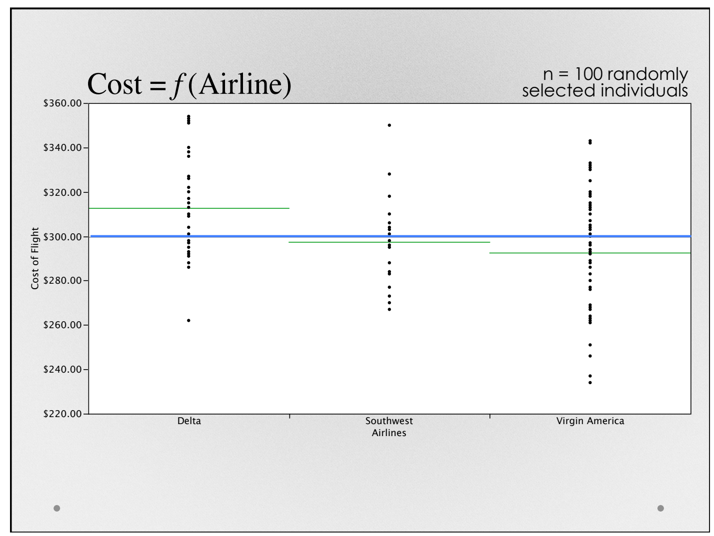
Individual's Score on Outcome Measure = Overall Constant + Effect for Some Measured Variable + Individual Error

Price for Flight = $258 + $0.30 per minute of flight duration + Individual Error


Price for Flight
=
$258
+
$0.30 per minute of flight duration
+
Individual Error
Cost of Flight
Flight Duration (minutes)


n = 100 randomly selected individuals
Count

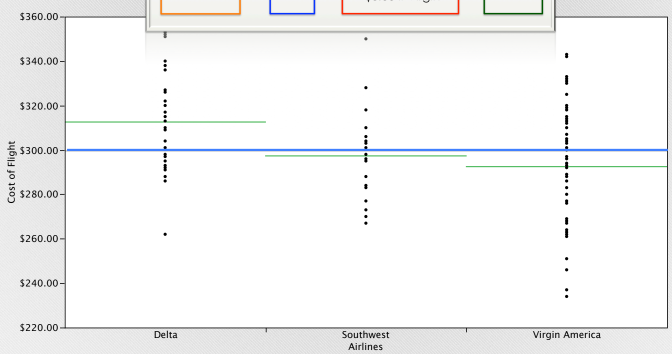

Cost = f(Airline)
n = 100 randomly selected individuals
Cost of Flight
Delta
Southwest Airlines
Virgin America


Cost = f(Airline)
n = 100 randomly selected individuals
Cost of Flight
Delta
Southwest Airlines
Virgin America

Individual's Score on Outcome Measure = Overall Constant + Effect for Some Measured Variable + Individual Error

Price for Flight = $300 + $11.80 if Delta / -$3.47 if Southwest / -$8.33 if Virgin + Individual Error

Price for Flight = $300 + $11.00 if Delta / -$3.47 if Southwest / -$8.33 if Virgin + Individual Error



Functional Relationships

Individual's Score on Outcome Measure = Overall Constant + Effect for Some Measured Variable

Statistical Relationships

Individual's Score on Outcome Measure = Overall Constant + Effect for Some Measured Variable + Individual Error

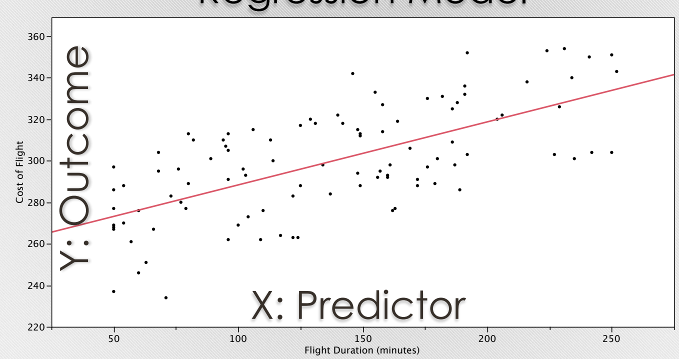
The General Linear Model

The General Linear Model

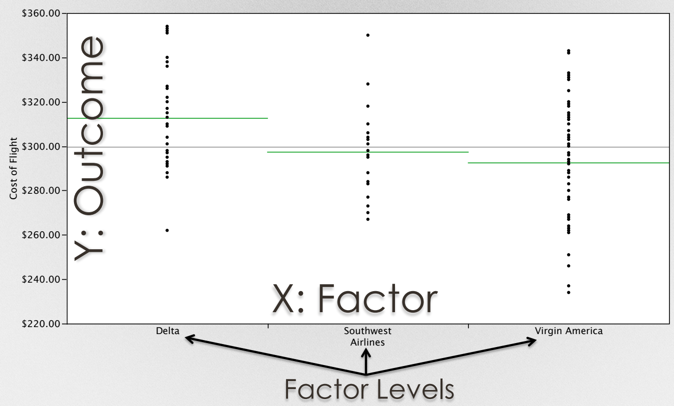
Regression Models

Mean Structure Models
(Analysis of Variance Models)

Regression Model



Mean Structure (ANOVA) Model



The General Linear Model

Regression Models

Mean Structure Models
(Analysis of Variance Models)

The General Linear Model

Regression Models
X Variable is Interval or Ratio Scaled

Mean Structure Models
(Analysis of Variance Models)
X Variable is Nominal or Ordinal Scaled

The General Linear Model

Regression Models
X Variable is Interval or Ratio Scaled

Mean Structure Models
(Analysis of Variance Models)
X Variable is Nominal or Ordinal Scaled

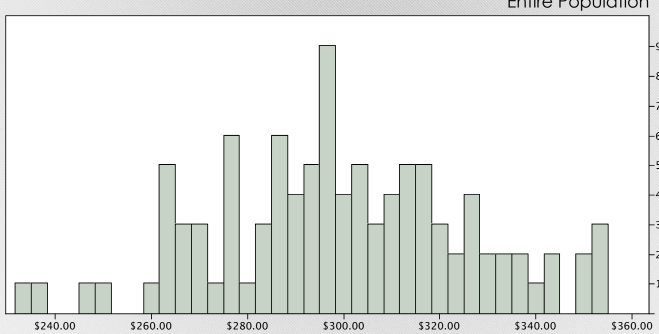
Population Model

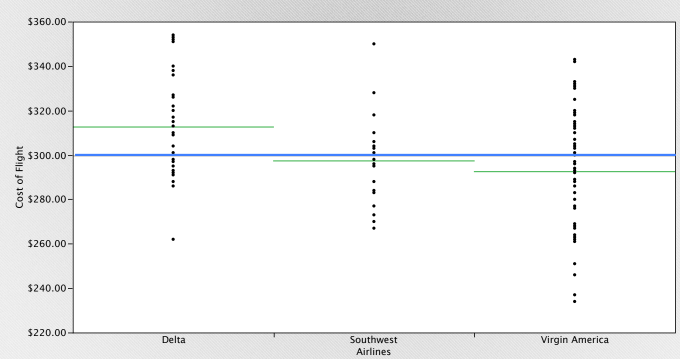
Sample Model

Population Model

Sample Model

N = 100
Entire Population





Price for Flight = \$300 + \$11.80 if Delta / -\$3.47 if Southwest / -\$8.33 if Virgin + Individual Error

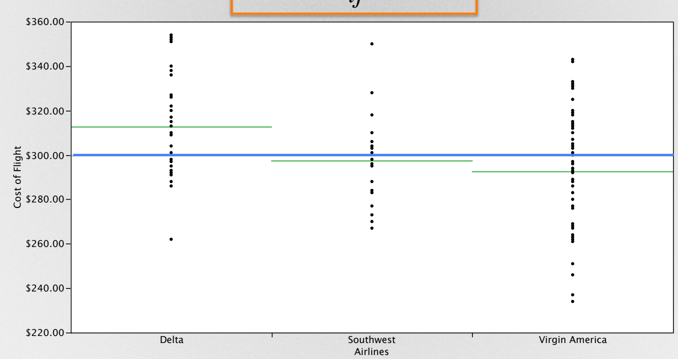
One Factor Linear Model
(Population Model)

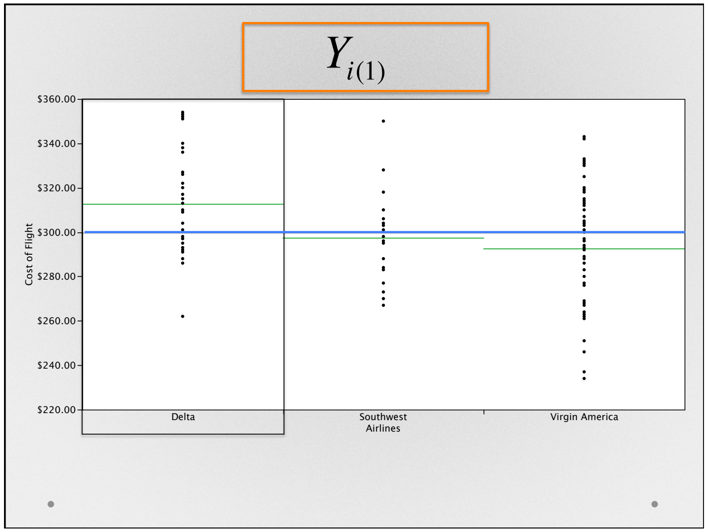
One Factor Linear Model
(Population Model)

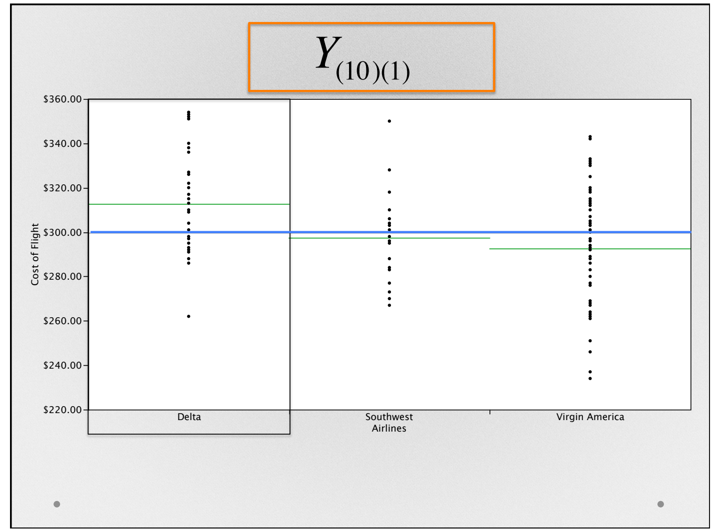
$$Y_{ij} =$$

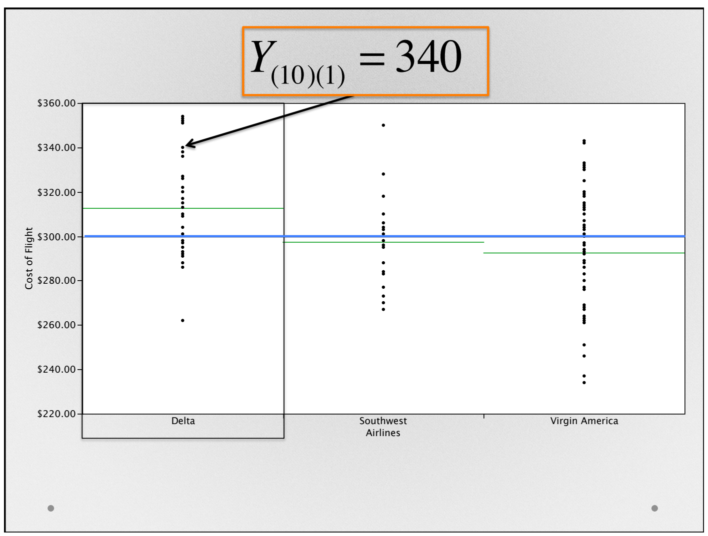
Score on Y for the ith individual in the jth group =

Y_{ij}









One Factor Linear Model
(Population Model)

$$\boxed{Y_{ij}} =$$

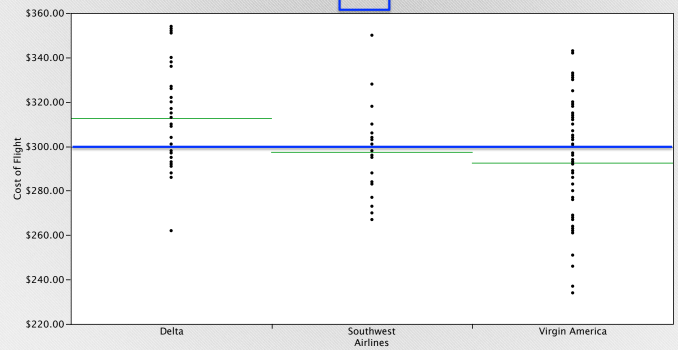
Score on Y for the ith individual in the jth group =

One Factor Linear Model
(Population Model)

$$\boxed{Y_{ij}} = \boxed{\mu}$$

Score on Y for the ith individual in the jth group = Grand Mean

$$\boxed{\mu}$$



One Factor Linear Model
(Population Model)

$$\boxed{Y_{ij}} = \boxed{\mu}$$

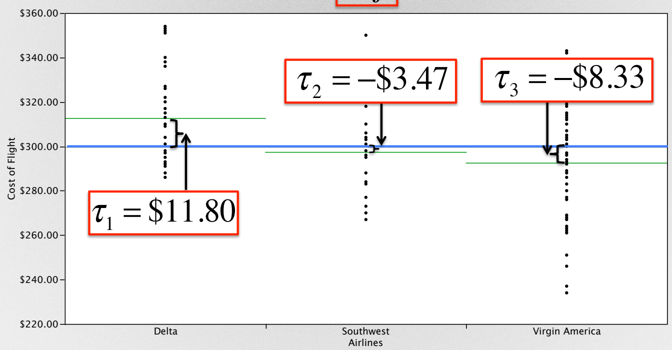
Score on Y for the ith individual in the jth group = Grand Mean

One Factor Linear Model
(Population Model)

$$\boxed{Y_{ij}} = \boxed{\mu} + \boxed{\tau_j}$$

Score on Y for the ith individual in the jth group = Grand Mean + Treatment Effect for Group j

$$\tau_j$$



Delta

$$\tau_1 = \$11.80$$

Southwest

$$\tau_2 = -\$3.47$$

Virgin America

$$\tau_3 = -\$8.33$$

One Factor Linear Model
(Population Model)

$$Y_{ij} = \mu + \tau_j$$

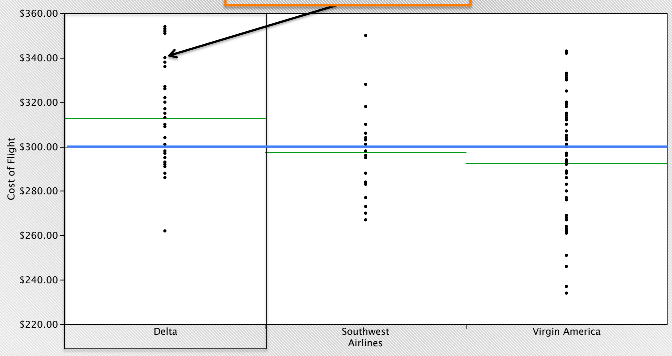
Score on Y for the ith individual in the jth group = Grand Mean + Treatment Effect for Group j

One Factor Linear Model
(Population Model)

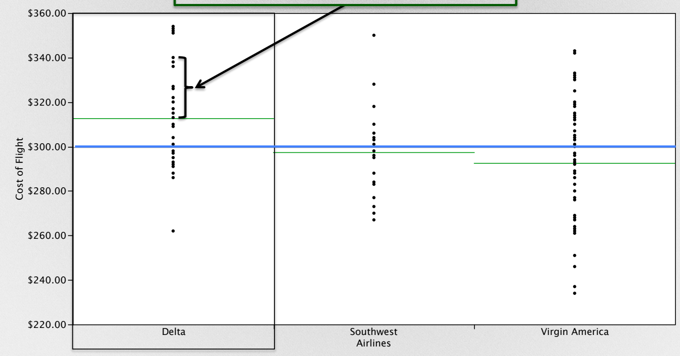
$$Y_{ij} = \mu + \tau_j + \varepsilon_{ij}$$

Score on Y for the ith individual in the jth group = Grand Mean + Treatment Effect for Group j + Error

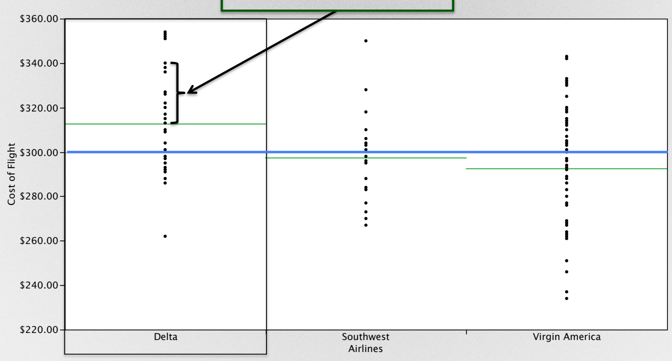
$$Y_{(10)(1)} = 340$$



$$\varepsilon_{(10)(1)} = 340 - 311.80$$



$$\varepsilon_{(10)(1)} = 28.2$$



One Factor Linear Model
(Population Model)

$$\boxed{Y_{ij}} = \boxed{\mu} + \boxed{\tau_j} + \boxed{\varepsilon_{ij}}$$

Score on Y for the ith individual in the jth group = Grand Mean + Treatment Effect for Group j + Error

For a Person on Delta

$$Y_i = \$300 + \$11.80 + \varepsilon_i$$

For a Person on Southwest

$$Y_i = \$300 - \$3.47 + \varepsilon_i$$

For a Person on Virgin America

$$Y_i = \$300 - \$8.33 + \varepsilon_i$$

One Factor Linear Model

(Population Model)

$$\boxed{Y_{ij}} = \boxed{\mu} + \boxed{\tau_j} + \boxed{\varepsilon_{ij}}$$

Score on Y for the ith individual in the jth group = Grand Mean + Treatment Effect for Group j + Error

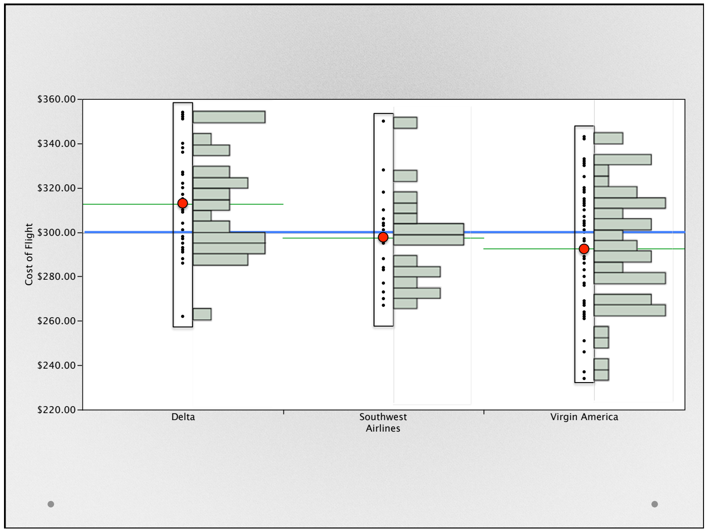
One Factor Linear Model

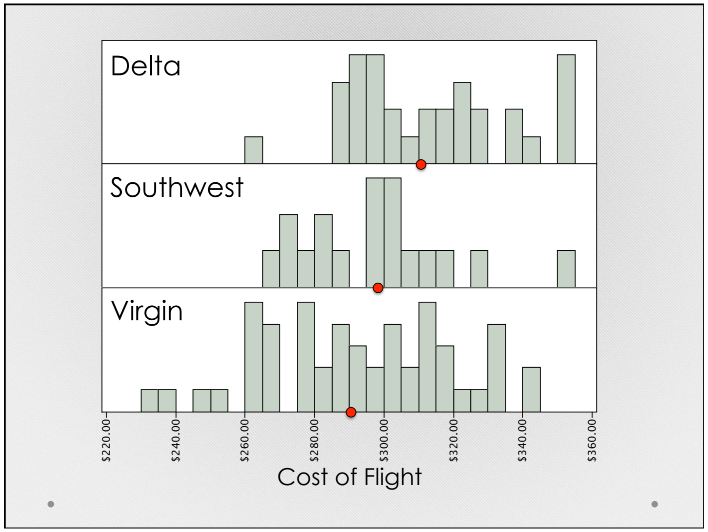
(Population Model)

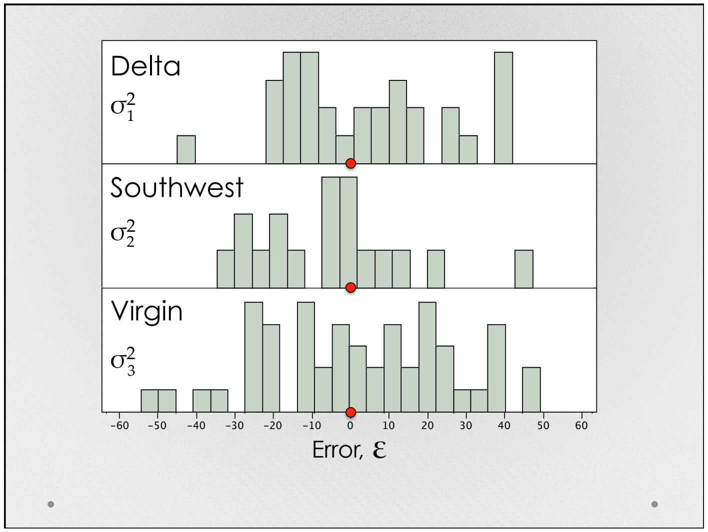
$$\boxed{Y_{ij}} = \boxed{\mu} + \boxed{\tau_j} + \boxed{\varepsilon_{ij}}$$

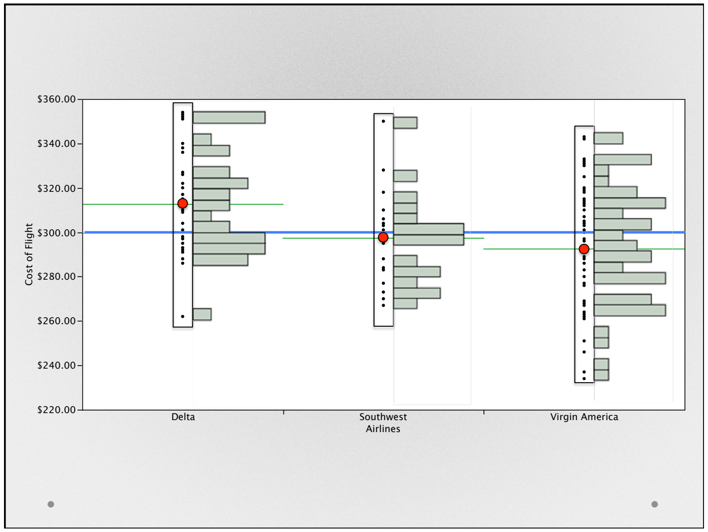
Score on Y for the ith individual in the jth group = Grand Mean + Treatment Effect for Group j + Error

The Distribution of ε


$360.00
$340.00
$320.00
$300.00
$280.00
$260.00
$240.00
$220.00
Cost of Flight
Delta
Southwest Airlines
Virgin America


Delta
Southwest
Virgin
$220.00
$240.00
$260.00
$280.00
$300.00
$320.00
$340.00
$360.00
Cost of Flight


Delta
σ_1^2
Southwest
σ_2^2
Virgin
σ_3^2
-60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60
Error, ε


$360.00
$340.00
$320.00
$300.00
$280.00
$260.00
$240.00
$220.00
Cost of Flight
Delta
Southwest Airlines
Virgin America

One Factor Linear Model
(Population Model)

$$\boxed{Y_{ij}} = \boxed{\mu} + \boxed{\tau_j} + \boxed{\varepsilon_{ij}}$$

Score on Y for the ith individual in the jth group = Grand Mean + Treatment Effect for Group j + Error

One Factor Linear Model
(Population Model)

$$\boxed{Y_{ij}} = \boxed{\mu} + \boxed{\tau_j} + \boxed{\varepsilon_{ij}}$$

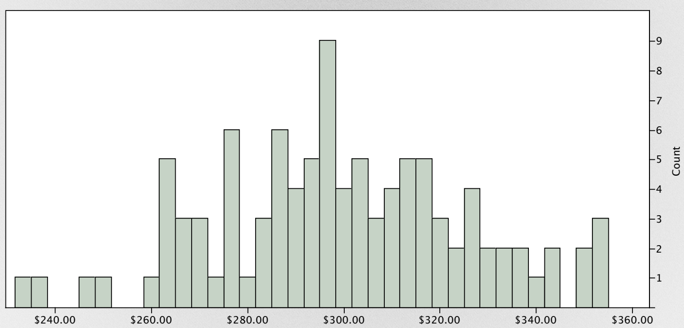
Score on Y for the ith individual in the jth group = Grand Mean + Treatment Effect for Group j + Error

Population Model

Sample Model

Population Model

Sample Model



One Factor Linear Model
(Sample Model)

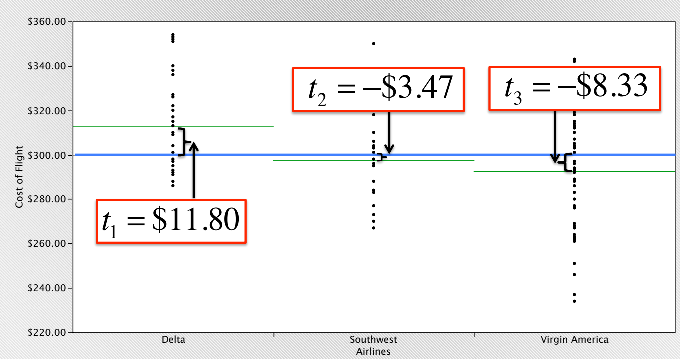
$$Y_{ij} = \bar{Y} + t_j + e_{ij}$$

Score on Y for the ith individual in the jth group = Est. Grand Mean + Estimated Treatment Effect for Group j + Error (Residual)

One Factor Linear Model
(Sample Model)

$$Y_{ij} = \bar{Y} + t_j + e_{ij}$$

Score on Y for the ith individual in the jth group = Est. Grand Mean + Estimated Treatment Effect for Group j + Error (Residual)



One Factor Linear Model
(Sample Model)

$$Y_{ij} = \bar{Y} + t_j + e_{ij}$$

Score on Y for the ith individual in the jth group = Est. Grand Mean + Estimated Treatment Effect for Group j + Error (Residual)

One Factor Linear Model
(Sample Model)

$$Y_{ij} = \bar{Y} + t_j + e_{ij}$$

Score on Y for the ith individual in the jth group = Est. Grand Mean + Estimated Treatment Effect for Group j + Error (Residual)

One Factor Linear Model
(Sample Model)

$$Y_{ij} = \bar{Y} + t_j + e_{ij}$$

Score on Y for the ith individual in the jth group = Est. Grand Mean + Estimated Treatment Effect for Group j + Error (Residual)

"Residual" Error

$$e_{ij} = Y_{ij} - \hat{Y}_{ij}$$

An individual's residual is the difference between that individual's ACTUAL observed value and the value predicted by the model

One Factor Linear Model
(Sample Model – Prediction for an Individual)

$$\boxed{\hat{Y}_{ij}} = \boxed{\bar{Y}} + \boxed{t_j}$$

Predicted Score on Y for the ith individual in the jth group = Est. Grand Mean + Estimated Treatment Effect for Group j

One Factor Linear Model
(Sample Model – Prediction for an Individual)

$$\boxed{\hat{Y}_{ij}} = \boxed{\bar{Y}_j}$$

Predicted Score on Y for the ith individual in the jth group = Sample Mean for Group j

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One Factor Linear Model
(Sample Model – Prediction for an Individual)

$$\boxed{\hat{Y}_{ij}} = \boxed{\bar{Y}_{\cdot j}}$$

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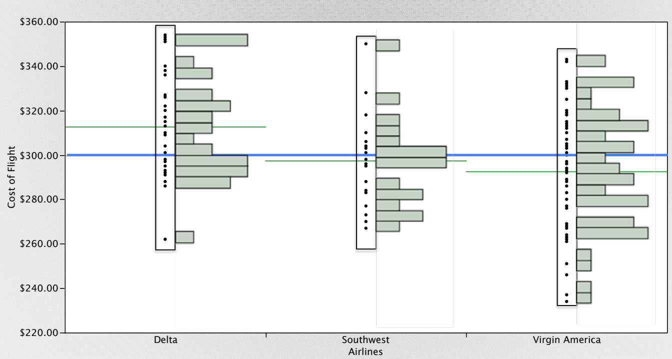

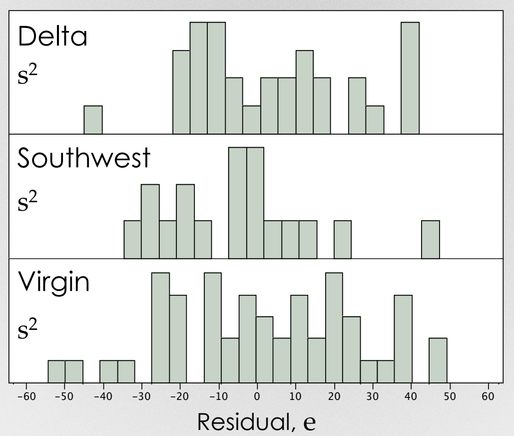

FROM BEFORE

Calculating the Variance

Population

$$SS = \Sigma(X_i - \mu)^2 \Rightarrow \sigma^2 = \frac{SS}{N}$$

Sample

$$SS = \Sigma(X_i - \bar{X})^2 \Rightarrow s^2 = \frac{SS}{n-1}$$

FROM BEFORE

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Sums of Squares for Error

$$SS_e = \sum_j \sum_i (Y_{ij} - \hat{Y}_{ij})^2$$

The sums of squares for error is the sum of each individual's squared error

Sums of Squares for Error

$$SS_e = \sum_j \sum_i e_{ij}^2$$

The sums of squares for error is the sum of each individual's squared error

Mean Squared Error

$$MS_e = s_p^2 = \frac{SS_e}{df_e}$$

The mean squared error is the mean of the squared error of each individual

Mean Squared Error Estimates σ^2

$$MS_e \mathrel{\hat{=}} \sigma^2$$

The mean squared error is an unbiased estimator of the variance in the population

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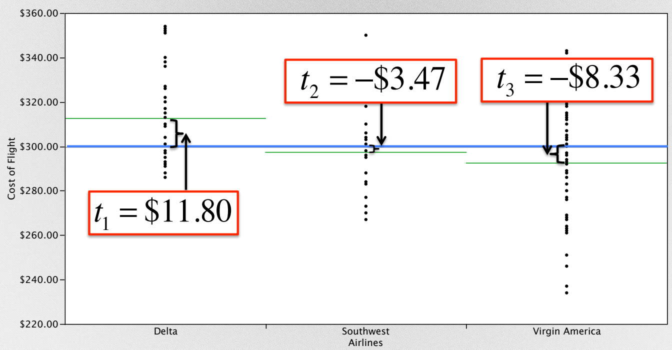
Sample Treatment Effects Estimate True Treatment Effects

$$t_j \hat{=} \tau_j$$

The treatment effects of a sample model are unbiased estimators of the true treatment effects in the population



Sample Treatment Effects Estimate True Treatment Effects

$$t_j \hat{=} \tau_j$$

The treatment effects of a Sample model are unbiased estimators of the true treatment effects in the population

Inferences about Treatment Effects

H_0: In the population the cost of flying is the same for these airlines

H_1: In the population the cost of flying is *not* the same for these airlines

Inferences about Treatment Effects

$$H_0 : \tau_1 = \tau_2 = ... = \tau_j = 0$$

$$H_1 : Not \text{ All } \tau_j = 0$$

Statistical Inference with Linear Models

- Analysis of Variance Approach
- General Linear Test Approach
  (model comparison)