


The Distribution  
of Sample Means



The Logic of  
Hypothesis Testing

● 1

Single Group Analyses		Single Group Experiments
T-test	Z-test	Single Sample Design
	T-test	Pre-Post Designs
Mixed-effects models		Repeated Measures Designs
Multi-Group Analyses		Multi-Group Experiments
	T-test	Two Group Design
Analysis of Variance		Multi-Group Design
Regression Analysis		Continuous Predictor Design
Mixed Analyses		Mixed Experiments
Mixed-effects models		Group Design with Repeated Measurements
Analysis of Covariance		Group Design also measured on a continuous variable

● 2

Single Group Analyses		Single Group Experiments
T-test	<b>Z-test</b>	<b>Single Sample Design</b>
	T-test	Pre-Post Designs
Mixed-effects models		Repeated Measures Designs
Multi-Group Analyses		Multi-Group Experiments
	T-test	Two Group Design
Analysis of Variance		Multi-Group Design
Regression Analysis		Continuous Predictor Design
Mixed Analyses		Mixed Experiments
Mixed-effects models		Group Design with Repeated Measurements
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● 3

## Z-Test Statistic

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

FROM SPSS

● 4

## Z-Test Statistic

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

- Requires knowledge of population  $\mu$  (mean before treatment)
- Requires knowledge of population  $\sigma^2$

• 5

Single Group Analyses		Single Group Experiments	
T-test	<b>Z-test</b>	<b>Single Sample Design</b>	
	T-test	Pre-Post Designs	
Mixed-effects models		Repeated Measures Designs	
Multi-Group Analyses		Multi-Group Experiments	
	T-test	Two Group Design	
Analysis of Variance		Multi-Group Design	
Regression Analysis		Continuous Predictor Design	
Mixed Analyses		Mixed Experiments	
Mixed-effects models		Group Design with Repeated Measurements	
Analysis of Covariance		Group Design also measured on a continuous variable	

• 6

Single Group Analyses		Single Group Experiments	
T-test	<b>Z-test<sub>αμ</sub></b>	<b>Single Sample Design</b>	
	T-test	Pre-Post Designs	
Mixed-effects models		Repeated Measures Designs	
Multi-Group Analyses		Multi-Group Experiments	
	T-test	Two Group Design	
Analysis of Variance		Multi-Group Design	
Regression Analysis		Continuous Predictor Design	
Mixed Analyses		Mixed Experiments	
Mixed-effects models		Group Design with Repeated Measurements	
Analysis of Covariance		Group Design also measured on a continuous variable	

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Single Group Analyses		Single Group Experiments	
<b>T-test<sub>μ</sub></b>	<b>Z-test<sub>αμ</sub></b>	<b>Single Sample Design</b>	
	T-test	Pre-Post Designs	
Mixed-effects models		Repeated Measures Designs	
Multi-Group Analyses		Multi-Group Experiments	
	T-test	Two Group Design	
Analysis of Variance		Multi-Group Design	
Regression Analysis		Continuous Predictor Design	
Mixed Analyses		Mixed Experiments	
Mixed-effects models		Group Design with Repeated Measurements	
Analysis of Covariance		Group Design also measured on a continuous variable	

• 8

Single Group Analyses		Single Group Experiments	
<b>T-test<sub>μ</sub></b> Z-test <sub>σμ</sub> <b>T-test</b> Mixed-effects models		<b>Single Sample Design</b> <b>Pre-Post Designs</b> Repeated Measures Designs	
Multi-Group Analyses		Multi-Group Experiments	
T-test Analysis of Variance Regression Analysis		Two Group Design Multi-Group Design Continuous Predictor Design	
Mixed Analyses		Mixed Experiments	
Mixed-effects models  Analysis of Covariance		Group Design with Repeated Measurements  Group Design also measured on a continuous variable	

Single Group Analyses		Single Group Experiments	
<b>T-test<sub>μ</sub></b> Z-test <sub>σμ</sub> <b>T-test</b> Mixed-effects models		<b>Single Sample Design</b> <b>Pre-Post Designs</b> Repeated Measures Designs	
Multi-Group Analyses		Multi-Group Experiments	
<b>T-test</b> Analysis of Variance Regression Analysis		<b>Two Group Design</b> Multi-Group Design Continuous Predictor Design	
Mixed Analyses		Mixed Experiments	
Mixed-effects models  Analysis of Covariance		Group Design with Repeated Measurements  Group Design also measured on a continuous variable	

## Z-Test Statistic

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

- Requires knowledge of population  $\mu$  (mean before treatment)
- **Requires knowledge of population  $\sigma^2$**

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sigma^2}{n}}$$



$$s^2 \hat{=} \sigma^2$$

The sample variance is an unbiased estimator of the population variance.

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Population

$$\sigma_{\bar{X}}$$

True  
Standard Error

Sample

$$s_{\bar{X}}$$

Estimated  
Standard Error

• 14

True Standard Error

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sigma^2}{n}}$$

• 15

$$s_{\bar{X}} = \frac{s}{\sqrt{n}} = \sqrt{\frac{s^2}{n}}$$

Estimated Standard Error

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## Z-Test Statistic

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

- Requires knowledge of population  $\mu$  (mean before treatment)
- **Requires knowledge of population  $\sigma^2$**

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## T-Test Statistic

$$t_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}}$$

- Requires knowledge of population  $\mu$  (mean before treatment)

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## T-Statistic, General Form

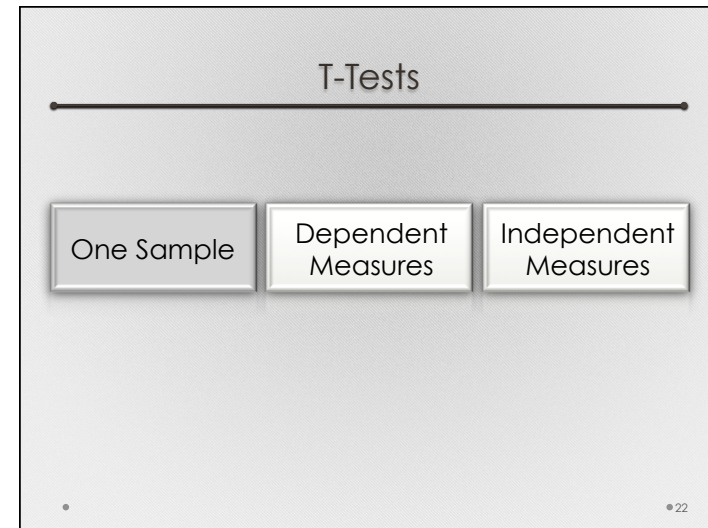
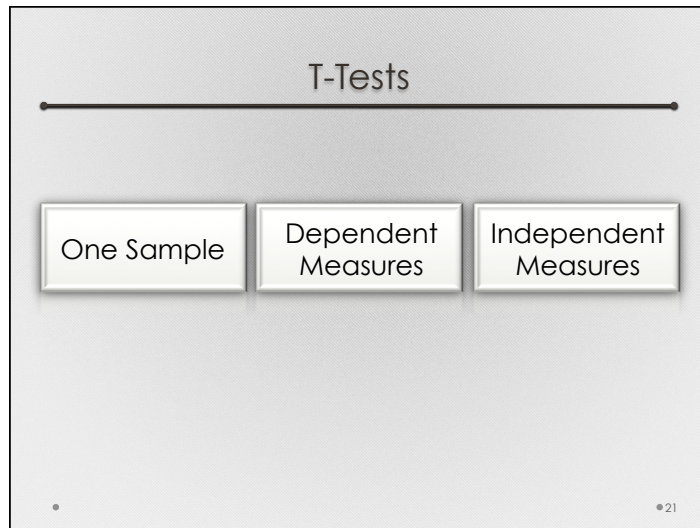
$$t = \frac{\text{sample statistic} - \text{population parameter}}{\text{estimated standard error of statistic}}$$

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## T-Statistic, General Form

$$t = \frac{\text{sample statistic} - \text{population parameter}}{\text{estimated standard error of statistic}}$$

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### T-Statistic, General Form

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$$t = \frac{\text{sample statistic} - \text{population parameter}}{\text{estimated standard error of statistic}}$$

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### T-Statistic, One Mean

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$$t = \frac{\text{sample mean} - \text{population mean}}{\text{estimated standard error of mean}}$$

• 24

## T-Statistic, One Mean

if  $H_0$  is True  
(if there is no effect)

$$t = \frac{\text{sample mean} - \text{population mean}}{\text{estimated standard error of mean}}$$

• 25

## T-Statistic, One Mean

$$t_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}}$$

- Requires knowledge of population  $\mu$   
(mean before treatment)

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## T-Tests

One Sample

Dependent Measures

Independent Measures

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## T-Tests

$$t_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}}$$

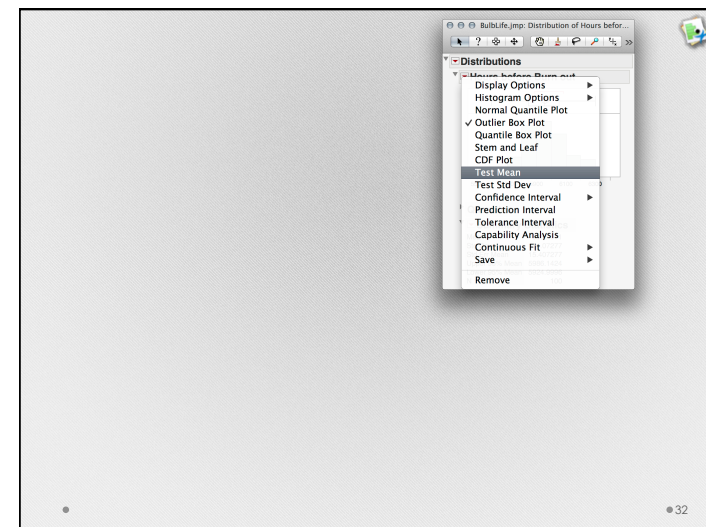
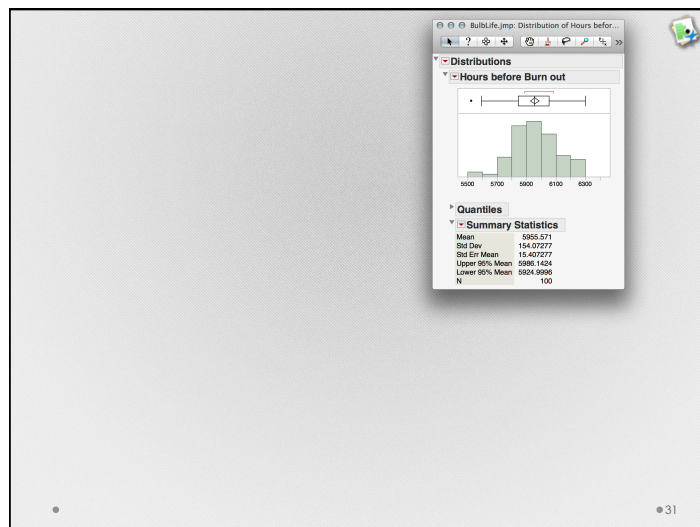
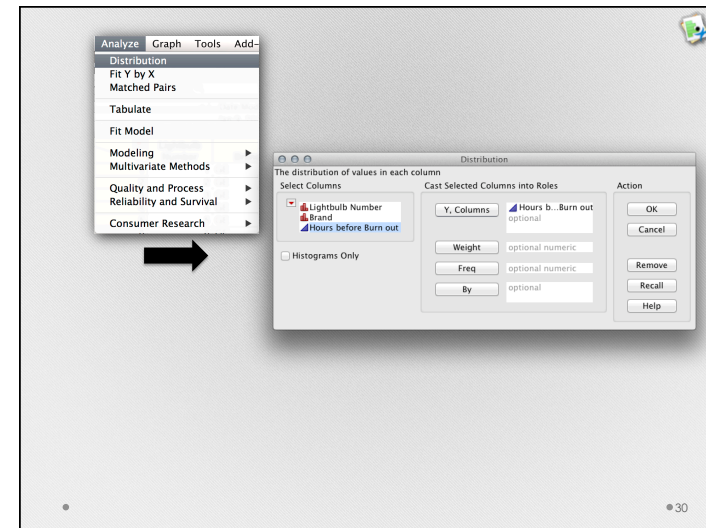
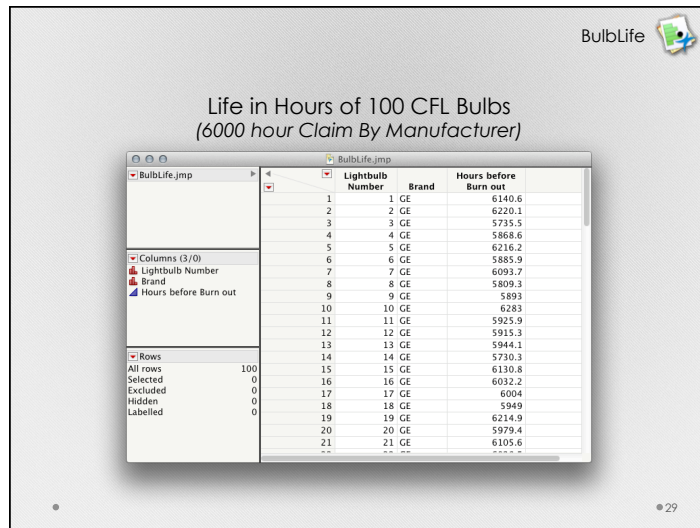
One Sample

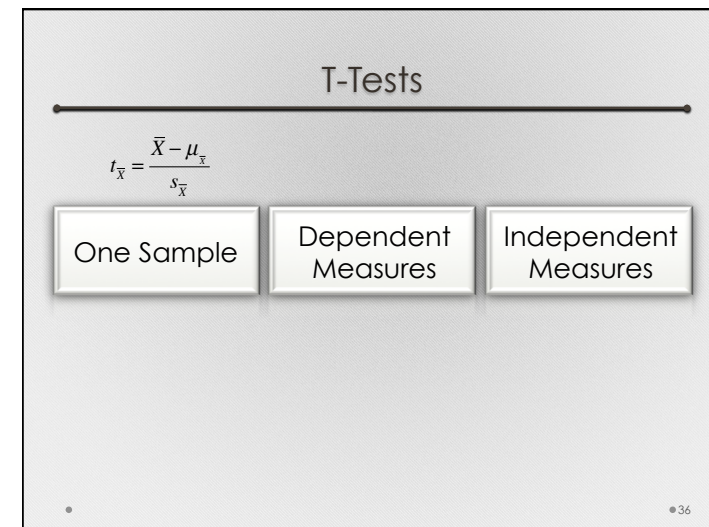
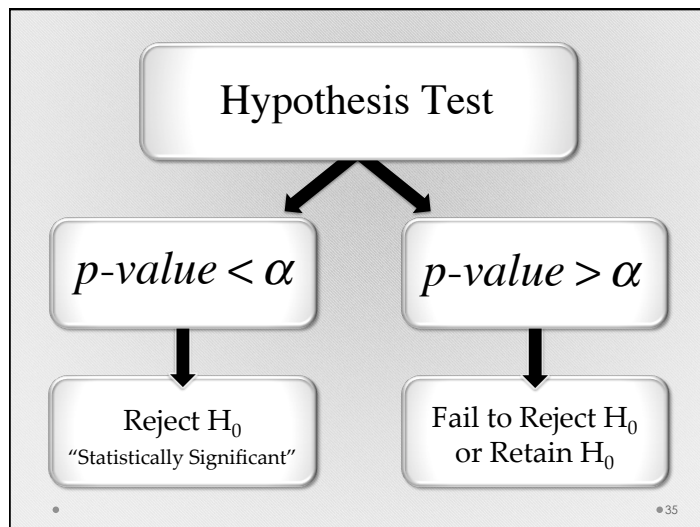
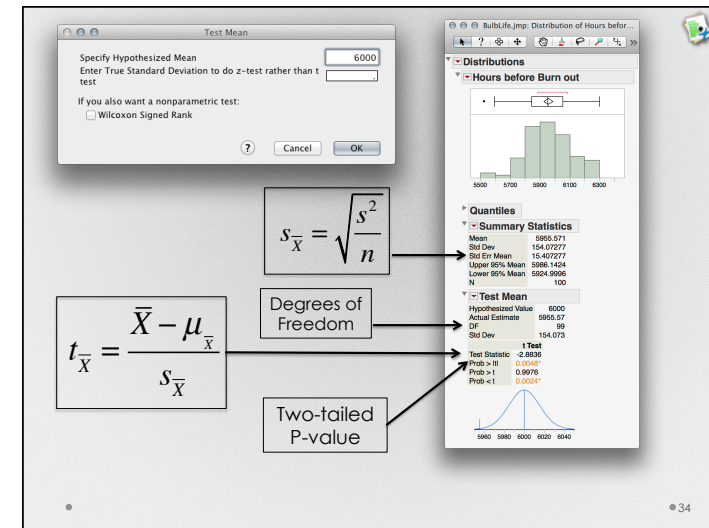
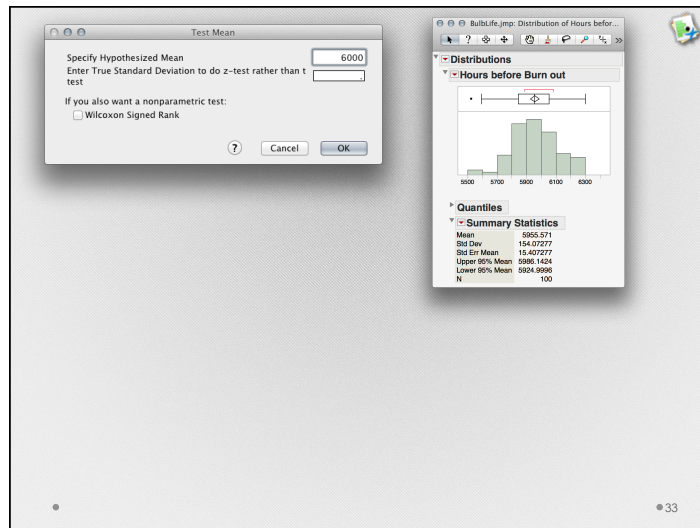
Dependent Measures

Independent Measures

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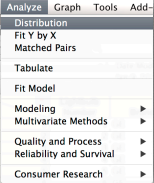
## T-Tests

$$t_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}}$$

One Sample

Dependent Measures

Independent Measures



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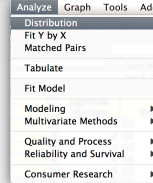
## T-Tests

$$t_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}}$$

One Sample

Dependent Measures

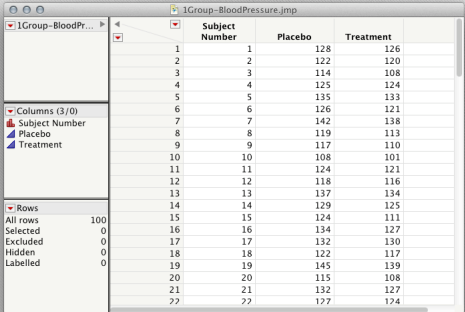
Independent Measures



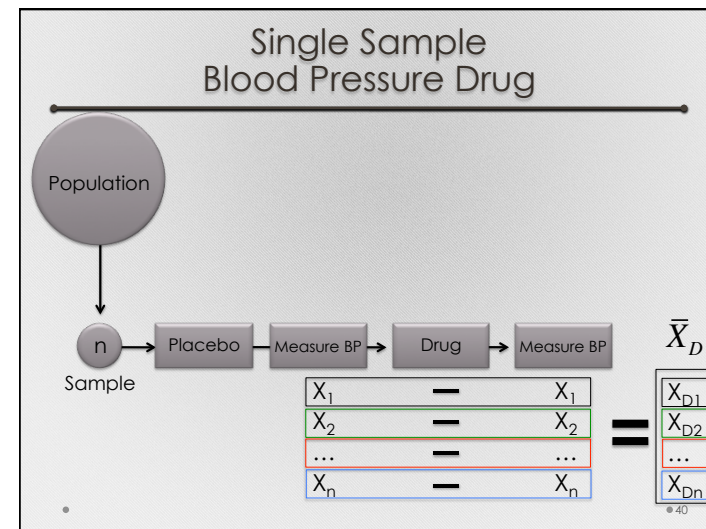
• 38

### Effects of Blood Pressure Drug Pre-Post, Dependent Measures

1 Group-BloodPressure



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## T-Statistic, General Form

$$t = \frac{\text{sample statistic} - \text{population parameter}}{\text{estimated standard error of statistic}}$$

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## T-Statistic, Dependent Measures

$$t = \frac{\text{sample difference score mean} - \text{population difference score mean}}{\text{estimated standard error of difference score mean}}$$

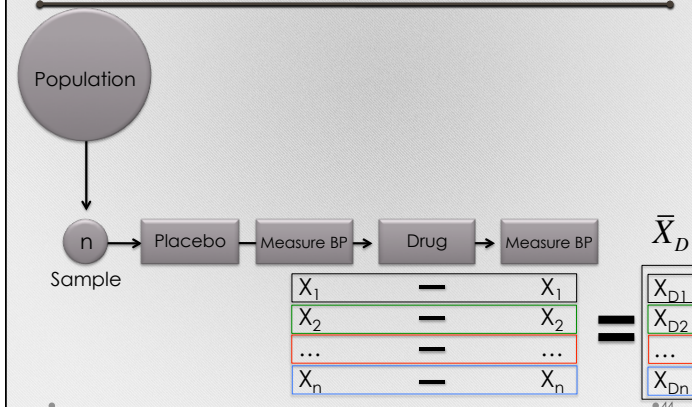
• 42

## T-Statistic, Dependent Measures

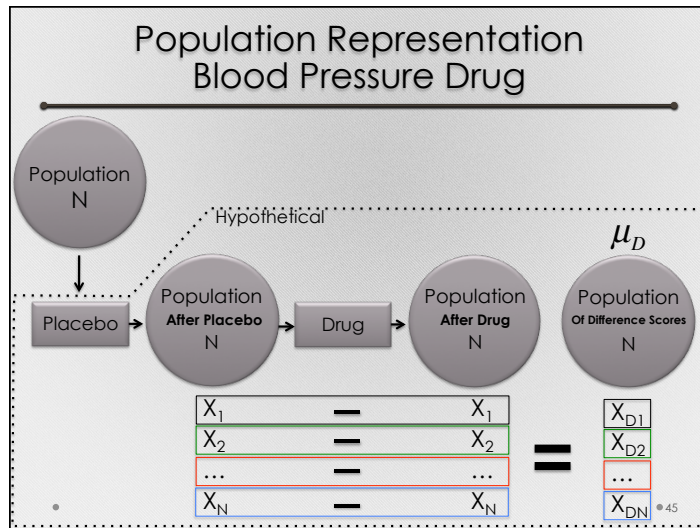
$$t = \frac{\bar{X}_D - \text{population difference score mean}}{\text{estimated standard error of difference score mean}}$$

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## Single Sample Blood Pressure Drug



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## T-Statistic, Dependent Measures

$$t = \frac{\bar{X}_D - \text{population difference score mean}}{\text{estimated standard error of difference score mean}}$$

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## T-Statistic, Dependent Measures

$$t = \frac{\bar{X}_D - \mu_D}{\text{estimated standard error of difference score mean}}$$

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## T-Statistic, Dependent Measures

$$t = \frac{\bar{X}_D - \mu_D}{s_{\bar{X}_D}}$$

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## T-Statistic, One Sample

$$t = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}}$$

• 49

## T-Statistic, Dependent Measures

$$t = \frac{\bar{X}_D - \mu_D}{s_{\bar{X}_D}}$$

• 50

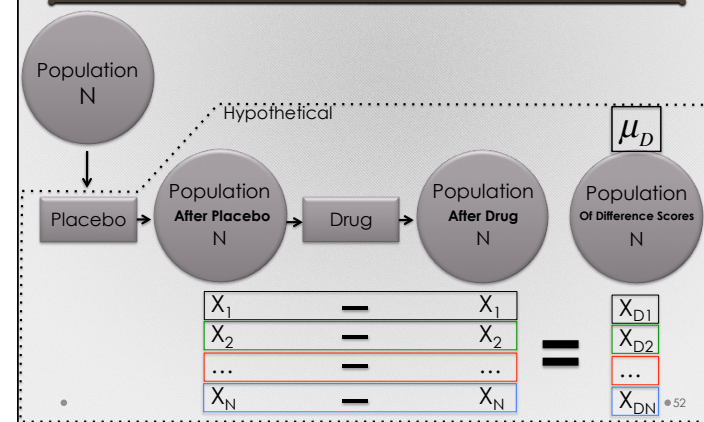
## T-Statistic, Dependent Measures

$$t = \frac{\bar{X}_D - \mu_D}{s_{\bar{X}_D}}$$

if  $H_0$  is True  
(if there is no effect)

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## Population Representation Blood Pressure Drug





### T-Statistic, Dependent Measures

if  $H_0$  is True  
(if there is no effect)

$$t = \frac{\bar{X}_D - \mu_D}{s_{\bar{X}_D}}$$

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### T-Statistic, Dependent Measures

if  $H_0$  is True  
(if there is no effect)

$$t = \frac{\bar{X}_D - 0}{s_{\bar{X}_D}}$$

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### T-Statistic, Dependent Measures

$$t_{\bar{X}_D} = \frac{\bar{X}_D}{s_{\bar{X}_D}}$$

• 55

### T-Statistic, Dependent Measures

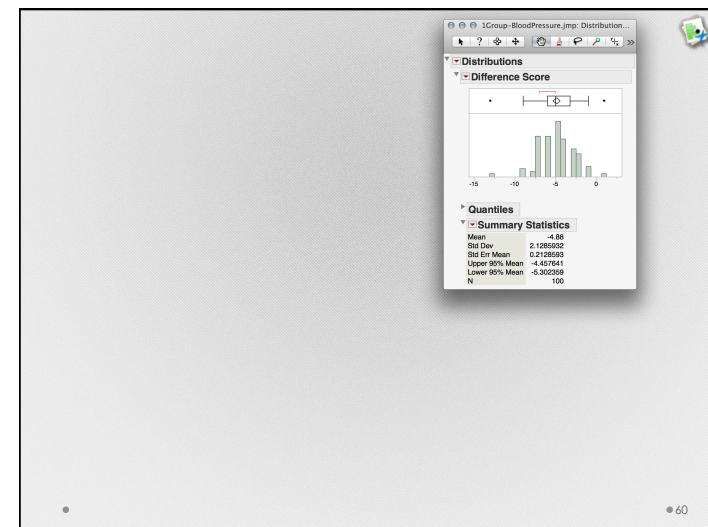
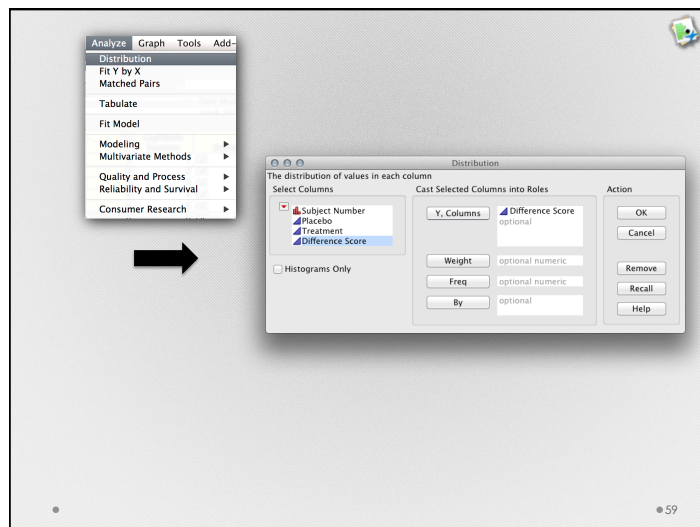
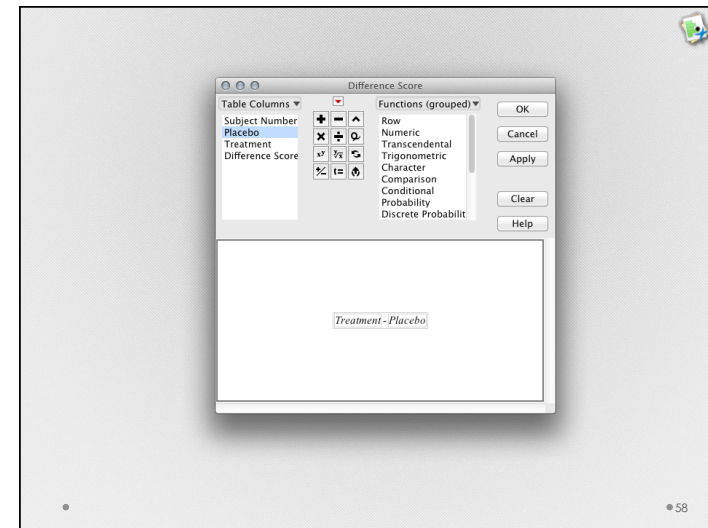
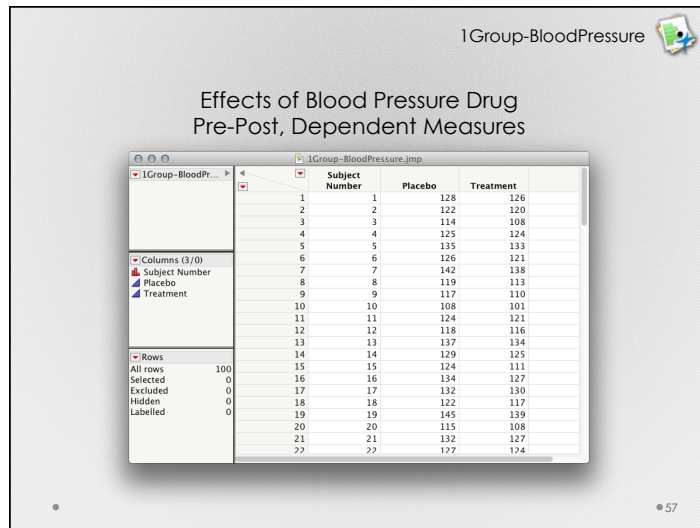
$$t_{\bar{X}_D} = \frac{\bar{X}_D}{s_{\bar{X}_D}}$$

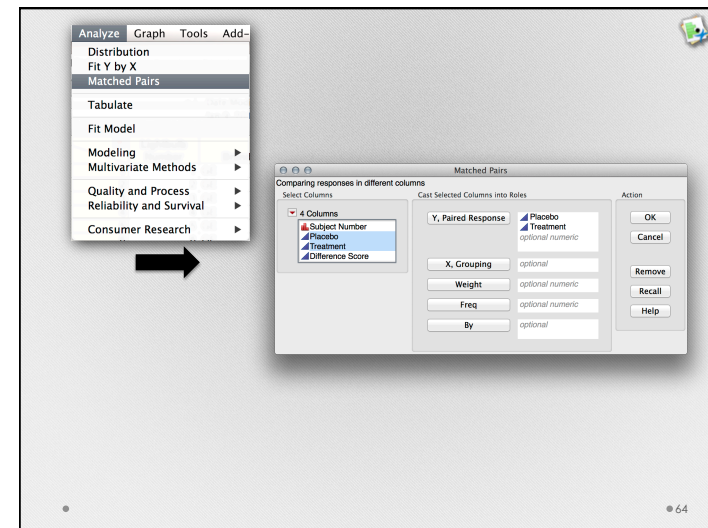
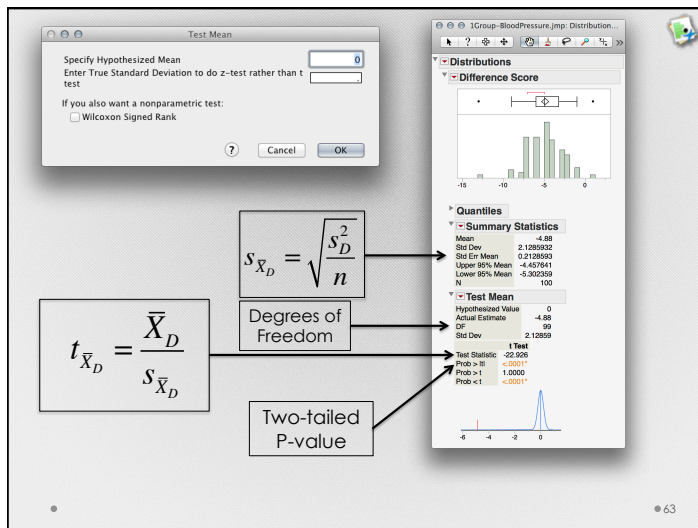
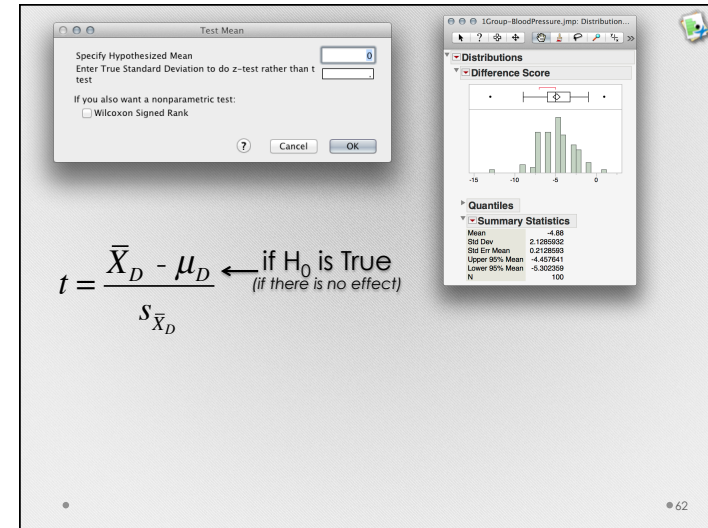
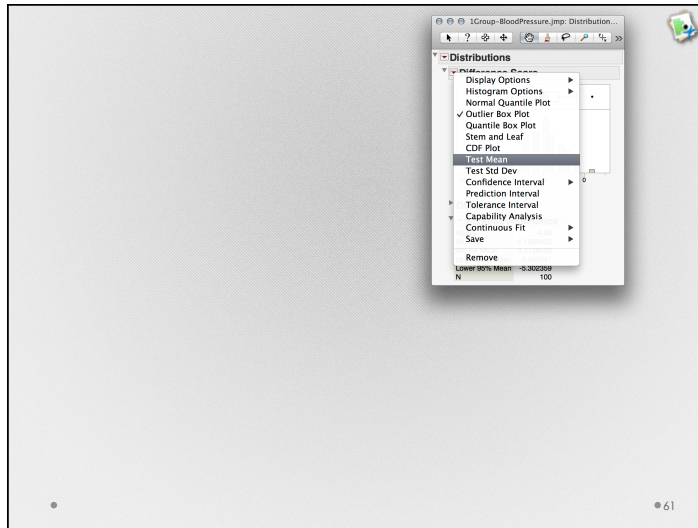
$$s_{\bar{X}_D} = \sqrt{\frac{s_D^2}{n}}$$

df for  $t_{\bar{X}_D} = (n - 1)$

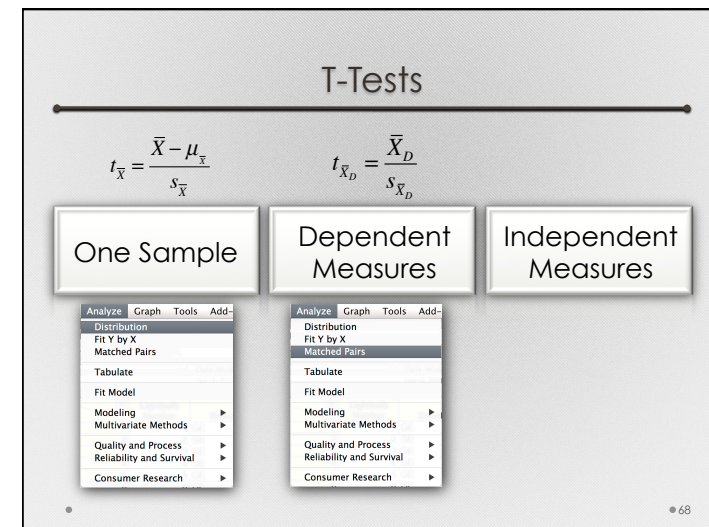
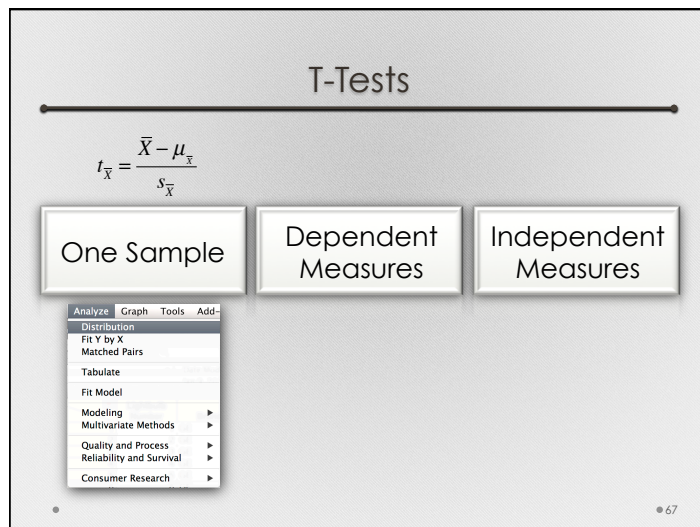
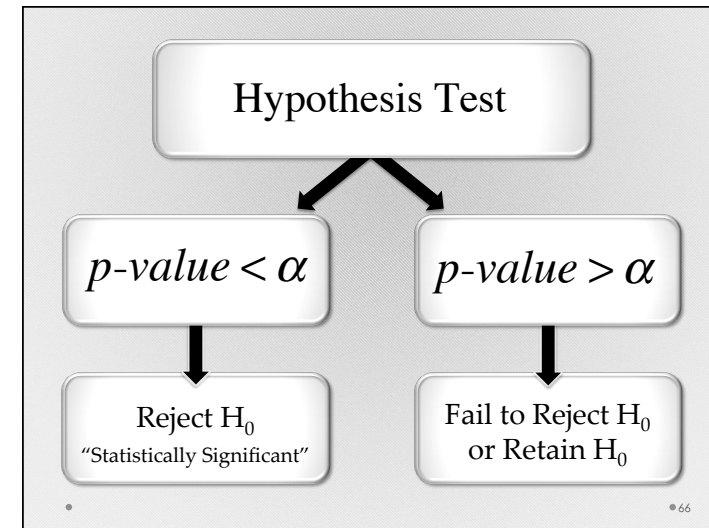
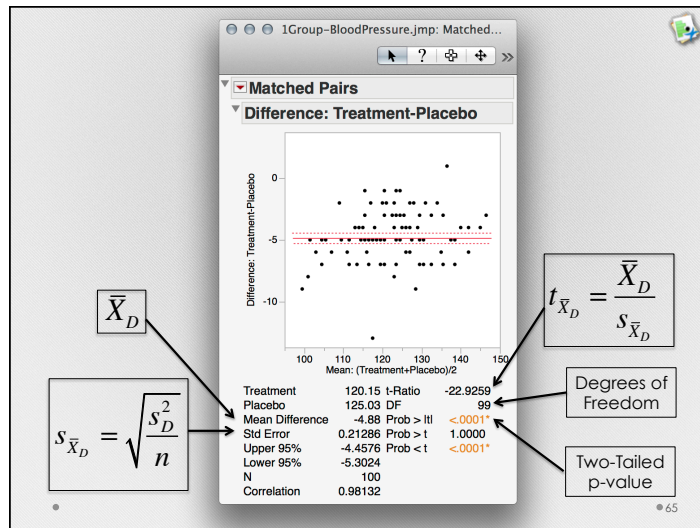
Where n is the number of **difference scores**

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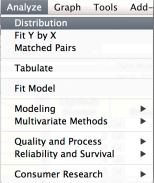




## T-Tests

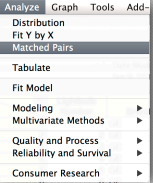
$$t_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{s_{\bar{X}}}$$

One Sample




$$t_{\bar{X}_D} = \frac{\bar{X}_D}{s_{\bar{X}_D}}$$

Dependent Measures



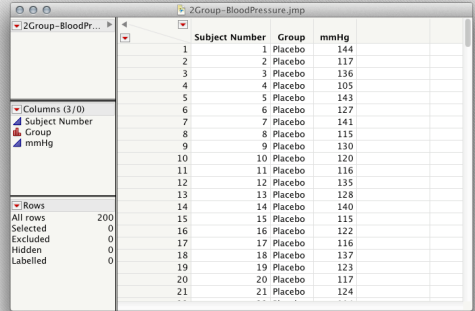
Independent Measures



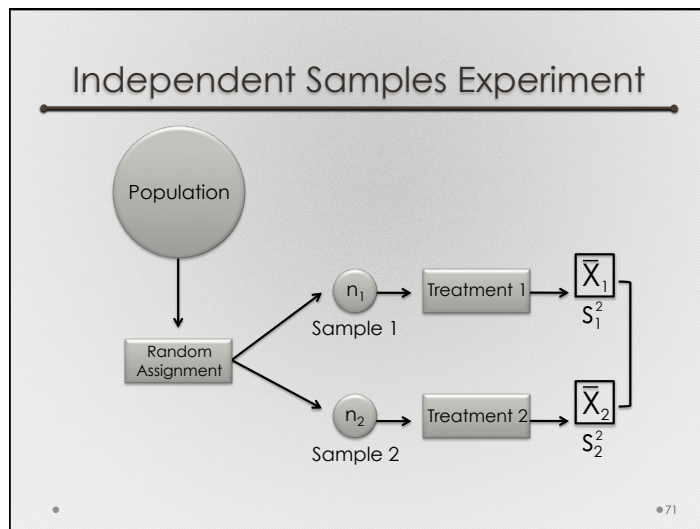
69

## Effects of Blood Pressure Drug Two Groups, Independent Measures

2Group-BloodPressure



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## T-Statistic, General Form

$$t = \frac{\text{sample statistic} - \text{population parameter}}{\text{estimated standard error of statistic}}$$

72

## T-Statistic, Two Mean Difference

$$t = \frac{\text{sample mean difference} - \text{population mean difference}}{\text{estimated standard error of the mean difference}}$$

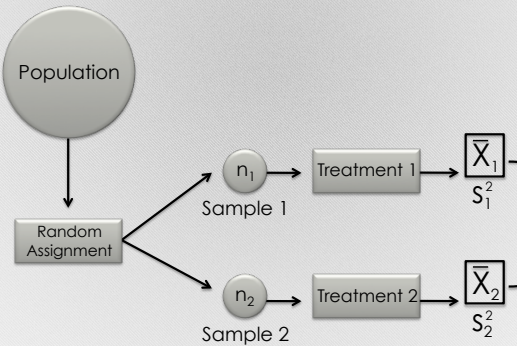
• 73

## T-Statistic, Two Mean Difference

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - \text{population mean difference}}{\text{estimated standard error of the mean difference}}$$

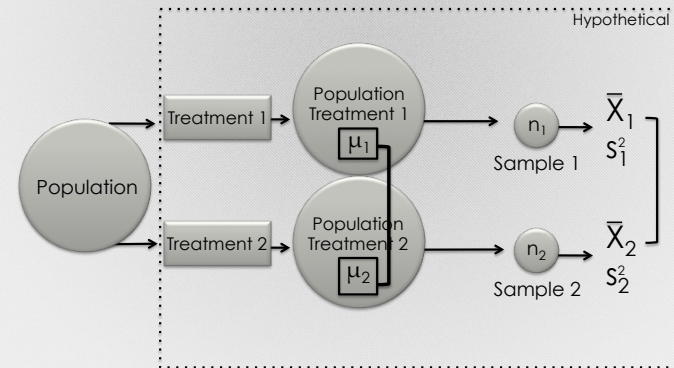
• 74

## Independent Samples Experiment



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## Independent Samples Experiment



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## T-Statistic, Two Mean Difference

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - \text{population mean difference}}{\text{estimated standard error of the mean difference}}$$

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## T-Statistic, Two Mean Difference

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\text{estimated standard error of the mean difference}}$$

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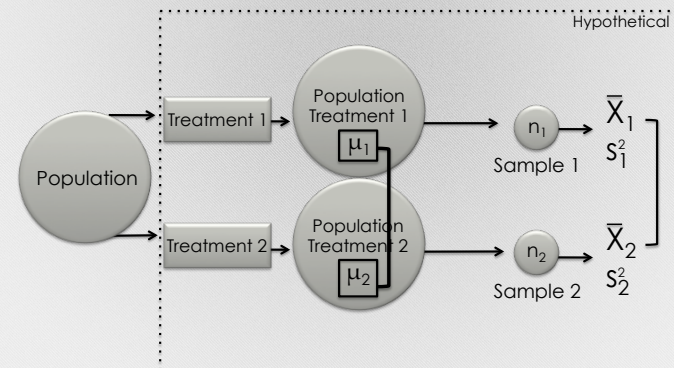
## T-Statistic, Two Mean Difference

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\text{estimated standard error of the mean difference}}$$

if  $H_0$  is True  
(if there is no effect)

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## Independent Samples Experiment



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If  $H_0$  is True  
 $(\mu_1 - \mu_2) = 0$

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## T-Statistic, Two Mean Difference

if  $H_0$  is True  
 (if there is no effect)

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\text{estimated standard error of the mean difference}}$$

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## T-Statistic, Two Mean Difference

if  $H_0$  is True  
 (if there is no effect)

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (0)}{\text{estimated standard error of the mean difference}}$$

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## T-Statistic, Two Mean Difference

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{\text{estimated standard error of the mean difference}}$$

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## T-Statistic, Two Mean Difference

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{\text{estimated standard error of the mean difference}}$$

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## T-Statistic, Two Mean Difference

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{(\bar{X}_1 - \bar{X}_2)}}$$

• 86

## T-Statistic, Two Mean Difference

$$t_{(\bar{X}_1 - \bar{X}_2)} = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{(\bar{X}_1 - \bar{X}_2)}}$$

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## T-Statistic, Two Mean Difference

$$t_{(\bar{X}_1 - \bar{X}_2)} = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{(\bar{X}_1 - \bar{X}_2)}}$$

↑  
Estimate of the Standard Deviation  
of the Sampling Distribution of the  
Sample Mean Difference

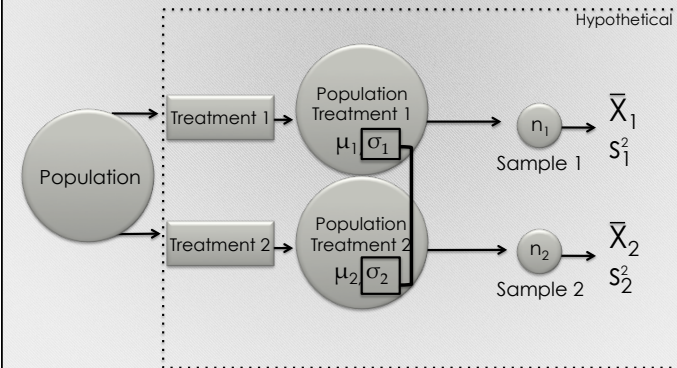
• 88

## Independent Samples t-test

- Appropriate for independent-samples or independent-measures designs
  - Treatments are applied to separate groups of people; no person experiences both treatments.
  - Also called a between-subjects design
- Two Varieties
  - Equal Variance Assumed (Pooled T-Test)
  - Equal Variance Not Assumed (Behrens-Fisher problem)

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## Independent Samples Experiment



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## Independent Samples t-test

- Appropriate for independent-samples or independent-measures designs
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  - Equal Variance Assumed (Pooled T-Test)
  - Equal Variance Not Assumed (Behrens-Fisher problem)

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## T-Statistic, Two Mean Difference

$$t_{(\bar{X}_1 - \bar{X}_2)} = \frac{(\bar{X}_1 - \bar{X}_2)}{s_{(\bar{X}_1 - \bar{X}_2)}}$$

$$df \text{ for } t_{(\bar{X}_1 - \bar{X}_2)} = (n_1 - 1) + (n_2 - 1)$$

Equal Variances Assumed

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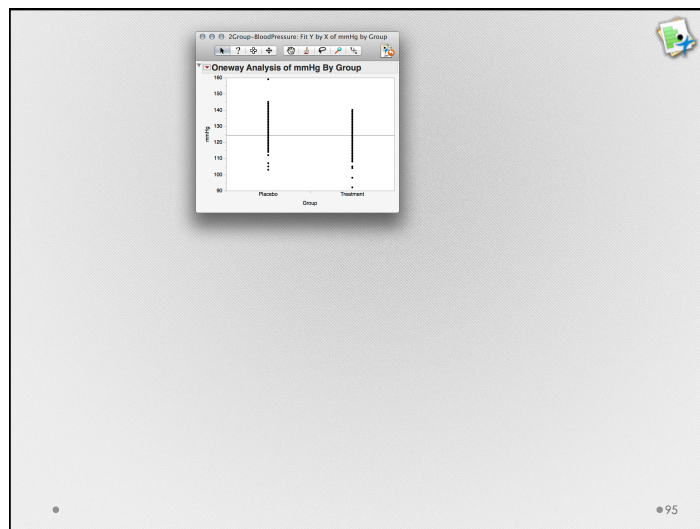
2Group-BloodPressure

### Effects of Blood Pressure Drug Two Groups, Independent Measures

Subject Number	Group	mmHg
1	Placebo	144
2	Placebo	117
3	Placebo	136
4	Placebo	105
5	Placebo	143
6	Placebo	127
7	Placebo	141
8	Placebo	115
9	Placebo	130
10	Placebo	120
11	Placebo	116
12	Placebo	135
13	Placebo	128
14	Placebo	140
15	Placebo	115
16	Placebo	122
17	Placebo	116
18	Placebo	137
19	Placebo	123
20	Placebo	117
21	Placebo	124

93

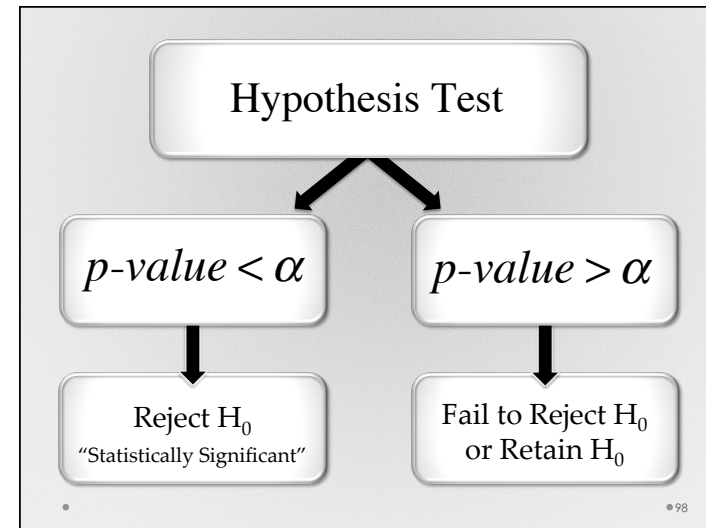
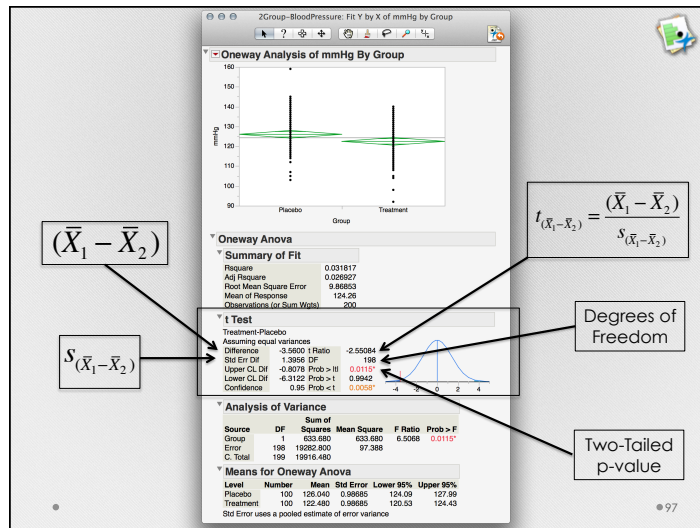
94



Equal Variance Assumed (Pooled T-Test)

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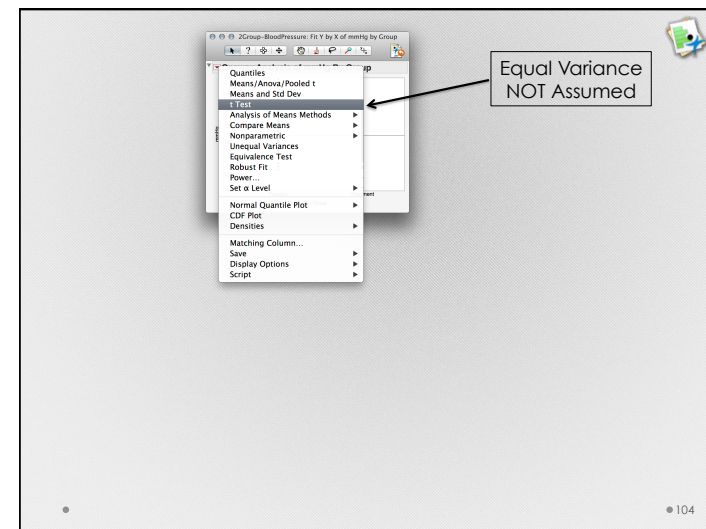
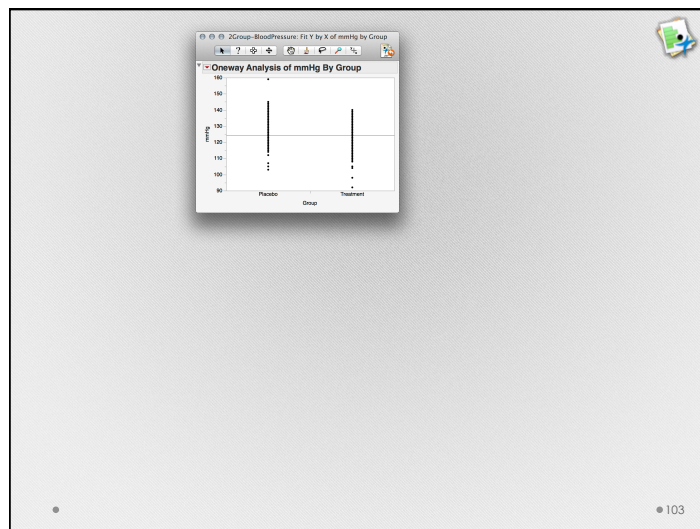
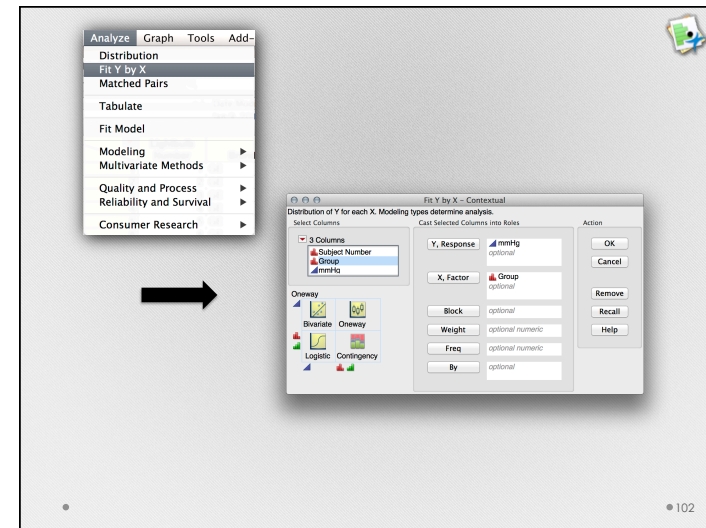
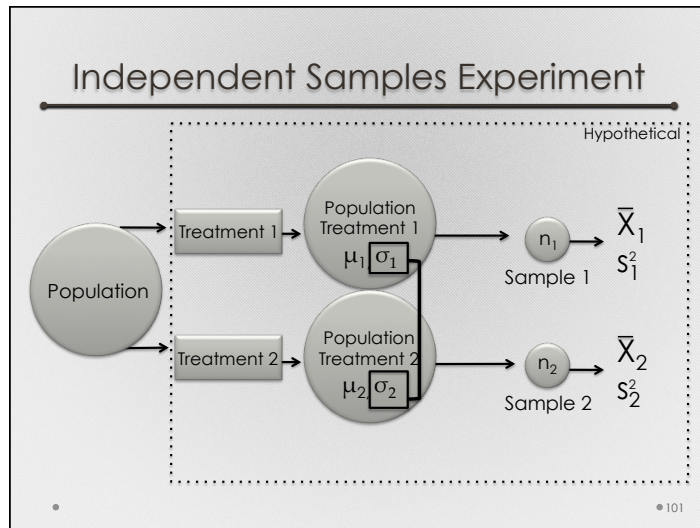


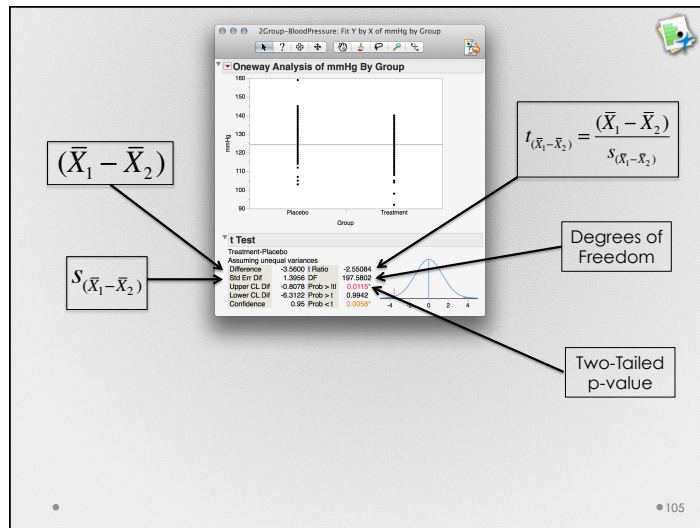
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- Appropriate for independent-samples or independent-measures designs
  - Treatments are applied to separate groups of people; no person experiences both treatments.
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- Two Varieties
  - Equal Variance Assumed (Pooled T-Test)
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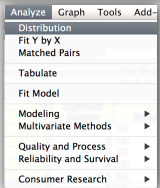




## T-Tests

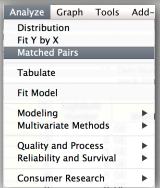
$$t_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{S_{\bar{X}}}$$

One Sample




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Dependent Measures



$$t_{(\bar{X}_1 - \bar{X}_2)} = \frac{(\bar{X}_1 - \bar{X}_2)}{S_{(\bar{X}_1 - \bar{X}_2)}}$$

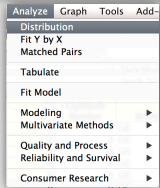
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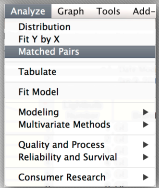
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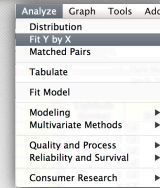
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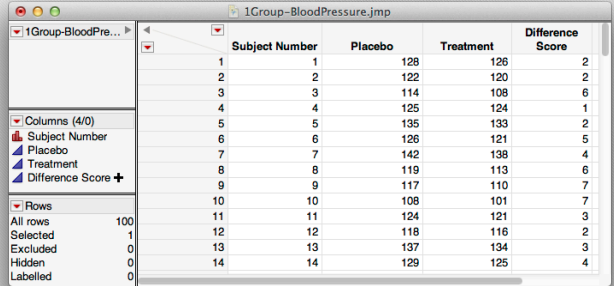
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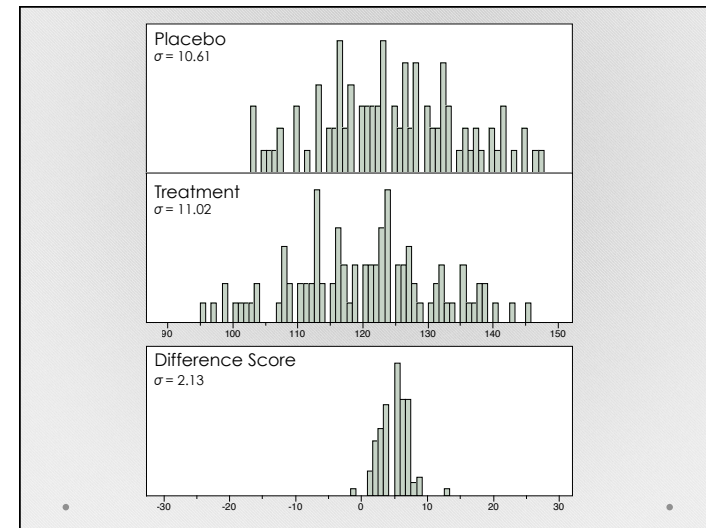
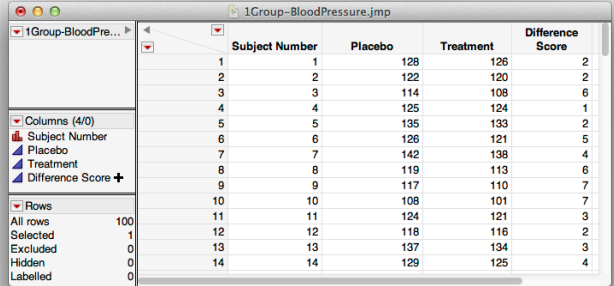
## Advantages of Repeated Measures

- Generally requires fewer subjects
  - Each person is providing two pieces of data
- Well suited for studying changes over time
  - A difference score focuses the question on individual change or growth
- Good for situations where there are large individual differences
  - Each person is their own "baseline"
  - More statistically powerful because the *difference scores* are less variable than the original scores





Subject Number	Placebo	Treatment	Difference Score
1	128	126	2
2	122	120	2
3	114	108	6
4	125	124	1
5	135	133	2
6	126	121	5
7	142	138	4
8	119	113	6
9	117	110	7
10	108	101	7
11	124	121	3
12	118	116	2
13	137	134	3
14	129	125	4

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## Problems with Repeated Measures

### Potential for carry-over and order effects

- Order effects: One treatment is always second, which means there could be some effect of practice, fatigue, time, etc
- Carry-over effects: One treatment can *leak* (carry-over) its effect into the other treatment

#### Solutions:

- Counterbalancing*: switch the order for half of the subjects. Will wash-out small order and carryover effects
- Use *Independent or matched samples*. If order or carryover effects are very strong, cannot use repeated-measures

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## Questions about the Effect of Some Treatment

