

	Truth	
	There is No Effect H_0 is True	There is an Effect H_1 is True
Decisions	Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ INCORRECT
	Reject H_0 ✗ INCORRECT	Reject H_0 ✓ CORRECT

	Truth	
	There is No Effect H_0 is True	There is an Effect H_1 is True
Decisions	Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ INCORRECT
	Reject H_0 ✗ Type I Error	Reject H_0 ✓ CORRECT

Type I Error

A Type I error is when *no effect is present*, but a researcher rejects the null hypothesis

"False Alarm" or "Alpha Error"

	Truth	
	There is No Effect H_0 is True	There is an Effect H_1 is True
Decisions	Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ INCORRECT
	Reject H_0 ✗ Type I Error	Reject H_0 ✓ CORRECT

	Truth	
	There is No Effect H_0 is True	There is an Effect H_1 is True
Decisions	Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ INCORRECT
	Reject H_0 ✗ Type I Error	Reject H_0 ✓ CORRECT

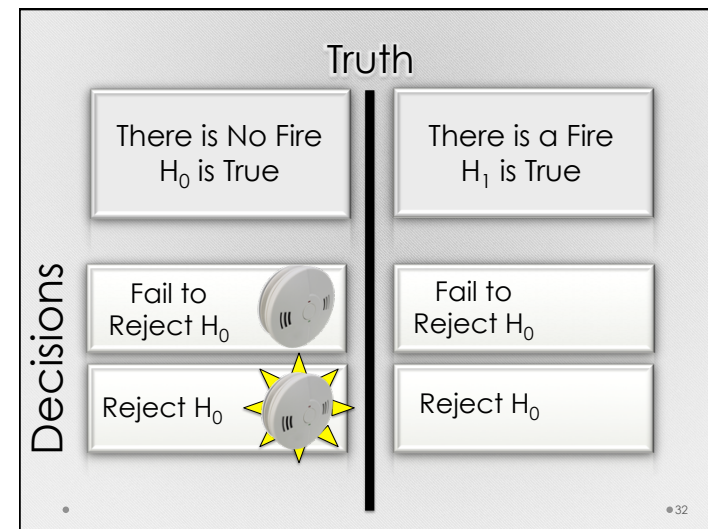
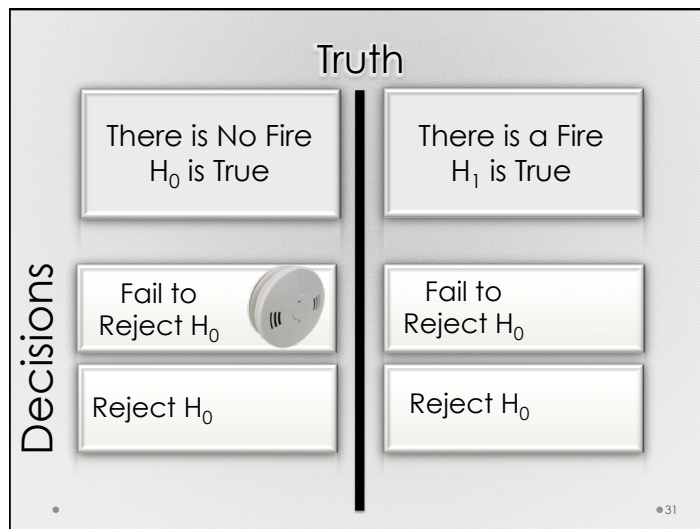
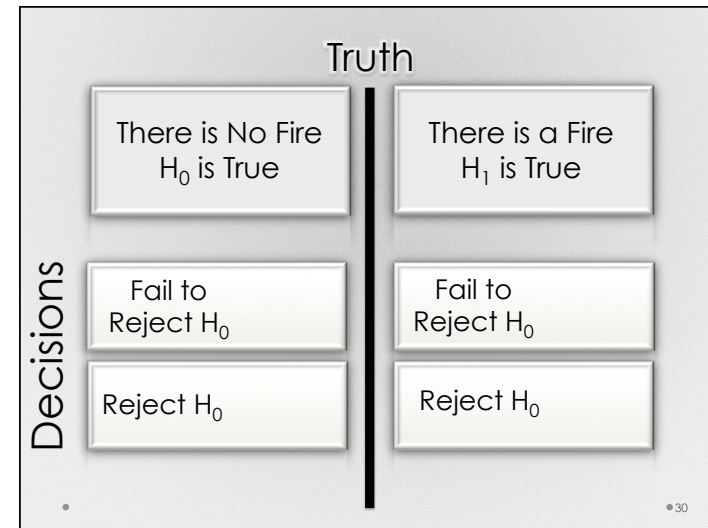
	Truth	
	There is No Effect H_0 is True	There is an Effect H_1 is True
Decisions	Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error
	Reject H_0 ✗ Type I Error	Reject H_0 ✓ CORRECT

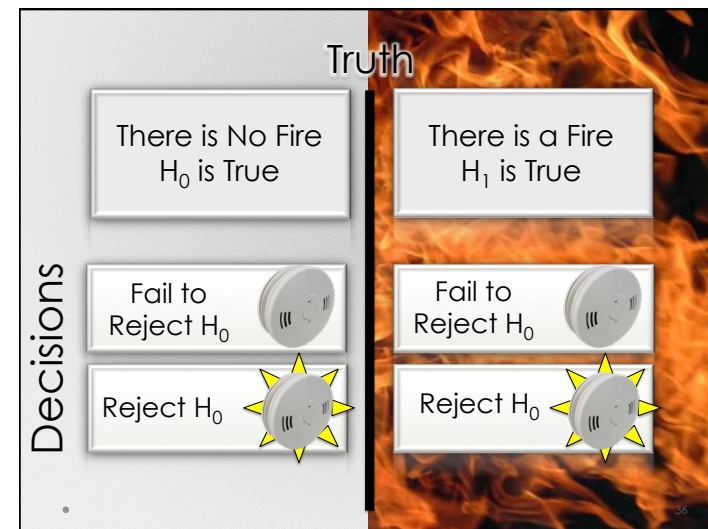
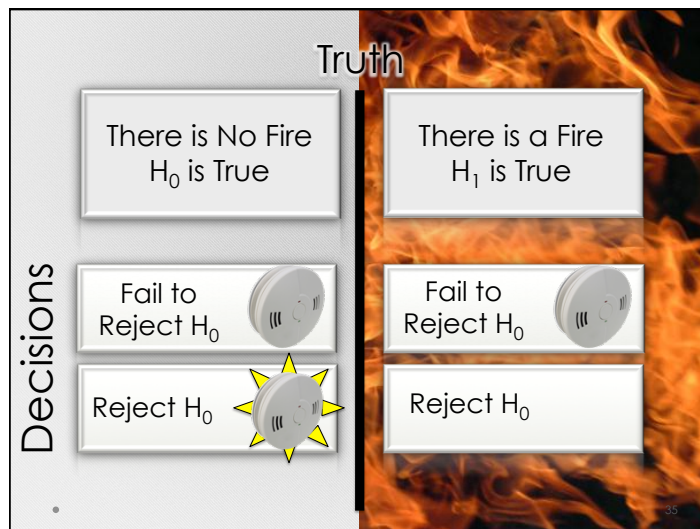
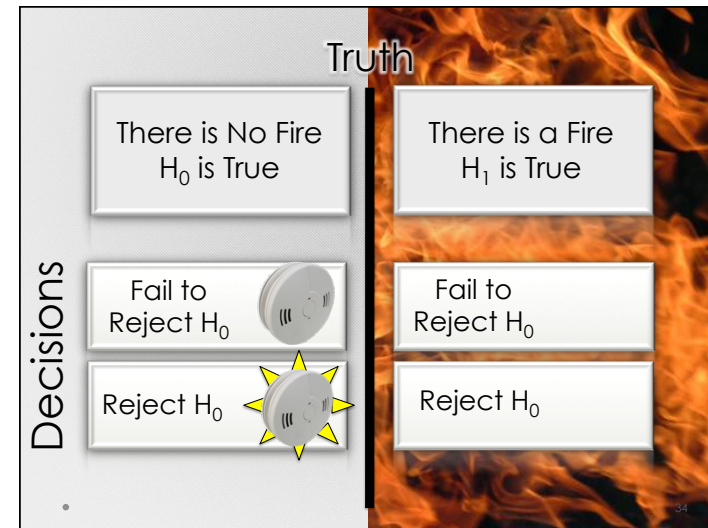
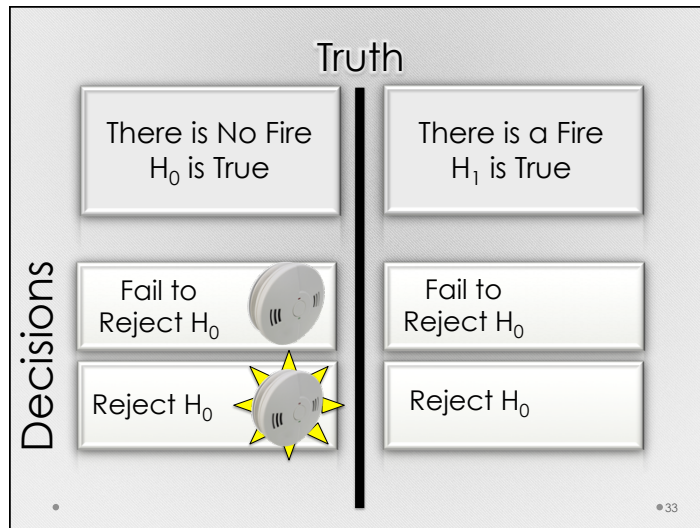
Type II Error

A Type II error is when a *real effect is present*, but a researcher *fails to reject* the null hypothesis

"Miss" or "Beta Error"

	Truth	
	There is No Effect H_0 is True	There is an Effect H_1 is True
Decisions	Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error
	Reject H_0 ✗ Type I Error	Reject H_0 ✓ CORRECT



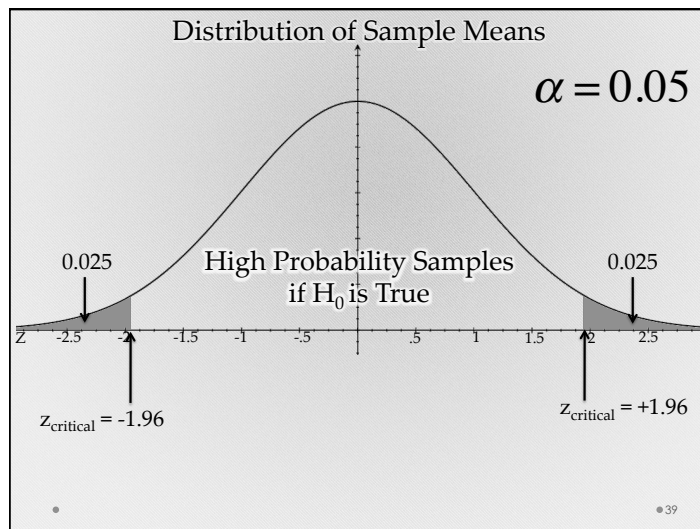


Truth	
There is No Effect H_0 is True	There is an Effect H_1 is True
Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error
Reject H_0 ✗ Type I Error	Reject H_0 ✓ CORRECT

Decisions

Truth	
There is No Effect H_0 is True	There is an Effect H_1 is True
Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error
Reject H_0 ✗ Type I Error	Reject H_0 ✓ CORRECT

Decisions




Truth	
There is No Effect H_0 is True	There is an Effect H_1 is True
Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error
Reject H_0 ✗ Type I Error	Reject H_0 ✓ CORRECT

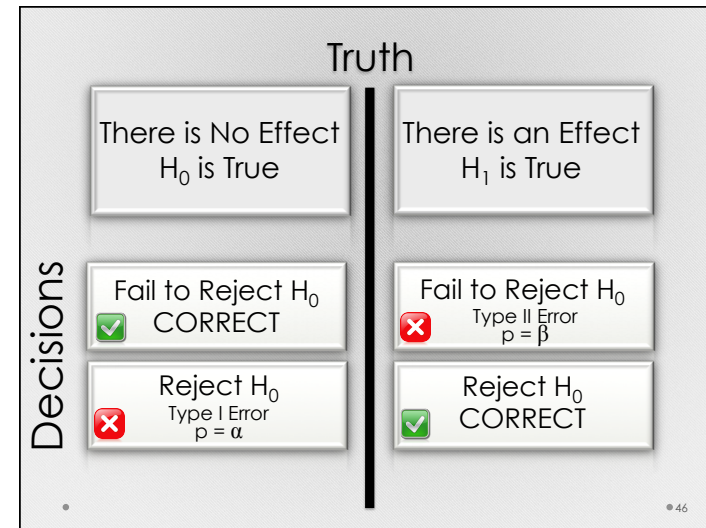
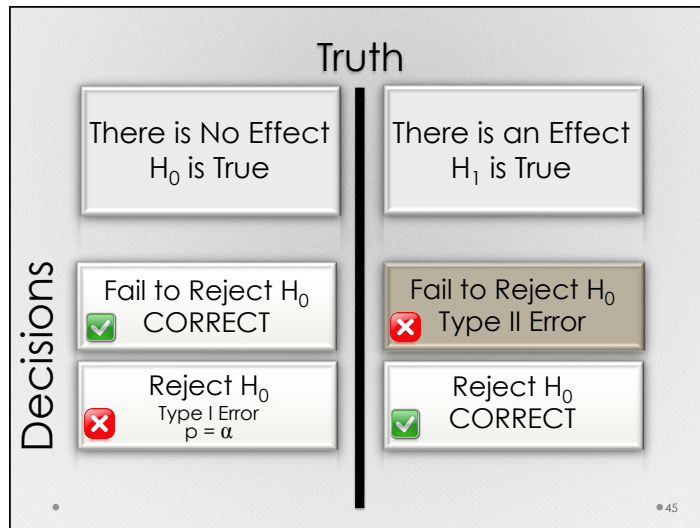
Decisions

	Truth	
	There is No Effect H_0 is True	There is an Effect H_1 is True
Decisions	Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error
	Reject H_0 ✗ Type I Error $p = \alpha$	Reject H_0 ✓ CORRECT

Probability of Type I Error
$p(\text{Type I Error if } H_0 \text{ True}) = \alpha$
$p(\text{Type I Error if } H_1 \text{ True}) = 0$

Probability of Type I Error
$p(\text{Type I Error} \mid H_0 \text{ True}) = \alpha$
$p(\text{Type I Error} \mid H_1 \text{ True}) = 0$
 Conditional Probability

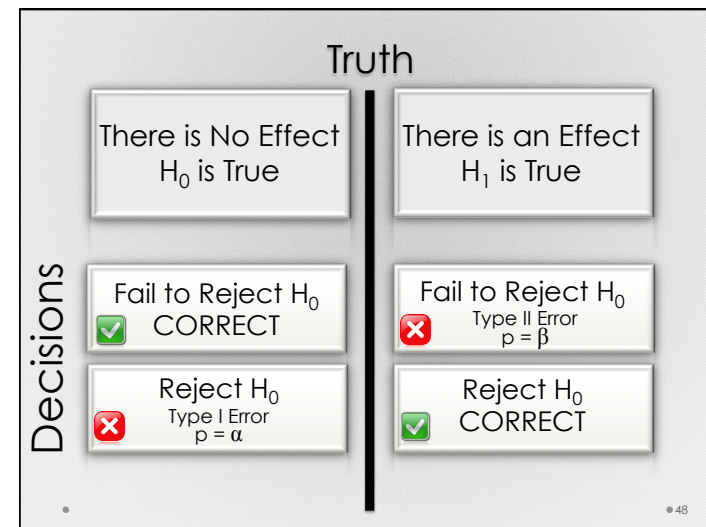
	Truth	
	There is No Effect H_0 is True	There is an Effect H_1 is True
Decisions	Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error
	Reject H_0 ✗ Type I Error $p = \alpha$	Reject H_0 ✓ CORRECT



Probability of Type II Error

$p(\text{Type II Error} \mid H_0 \text{ True}) = 0$
 $p(\text{Type II Error} \mid H_1 \text{ True}) = \beta$

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Truth	
There is No Effect H_0 is True	There is an Effect H_1 is True
Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error $p = \beta$
Reject H_0 ✗ Type I Error $p = \alpha$	Reject H_0 ✓ CORRECT

Decisions

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Truth	
There is No Effect H_0 is True	There is an Effect H_1 is True
Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error $p = \beta$
Reject H_0 ✗ Type I Error $p = \alpha$	Reject H_0 ✓ Statistical Power $p = 1 - \beta$

Decisions

• 50

Truth	
There is No Effect H_0 is True	There is an Effect H_1 is True
Fail to Reject H_0 ✓ CORRECT	Fail to Reject H_0 ✗ Type II Error $p = \beta$
Reject H_0 ✗ Type I Error $p = \alpha$	Reject H_0 ✓ Statistical Power $p = 1 - \beta$

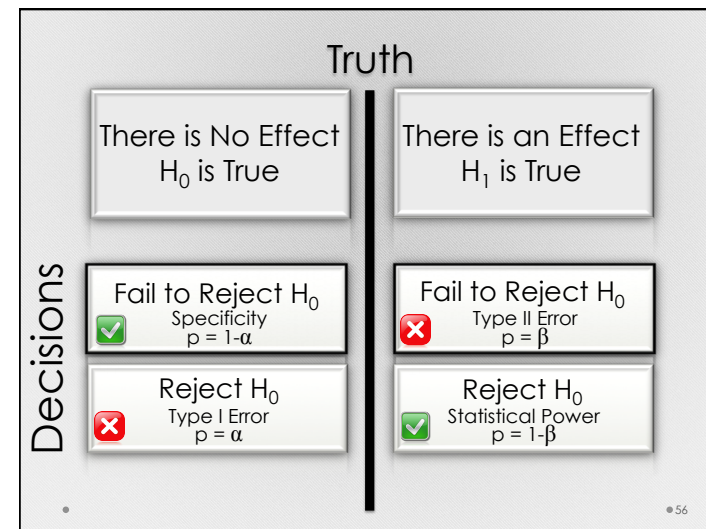
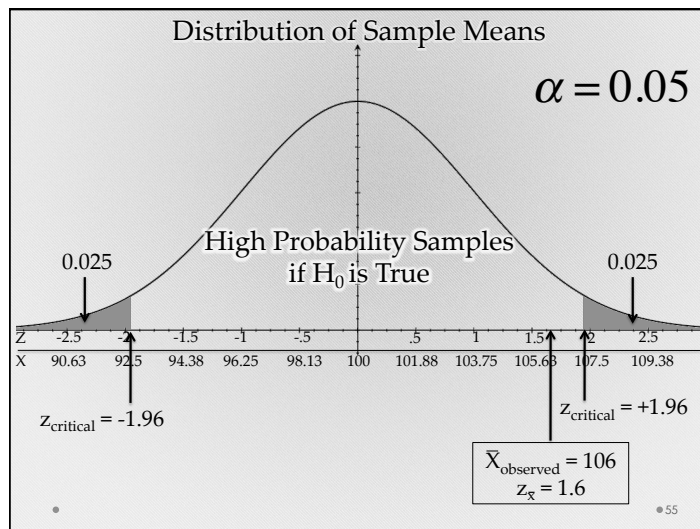
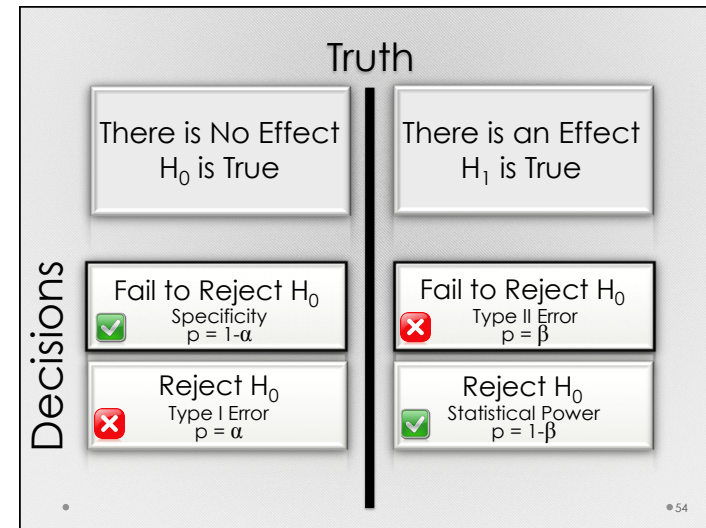
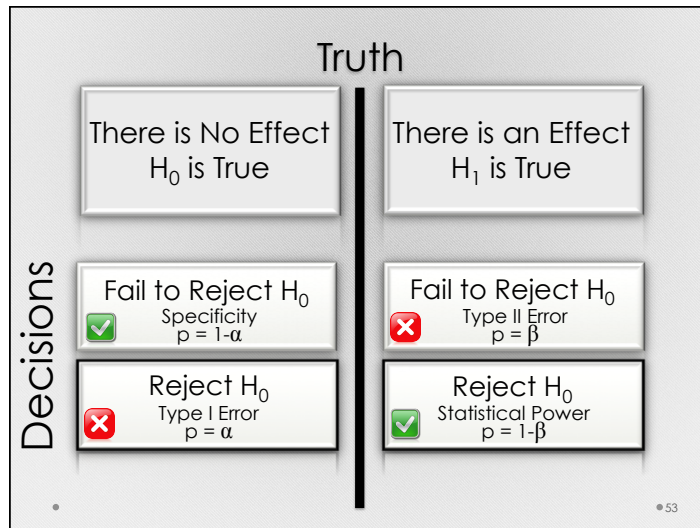
Decisions

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Truth	
There is No Effect H_0 is True	There is an Effect H_1 is True
Fail to Reject H_0 ✓ Specificity $p = 1 - \alpha$	Fail to Reject H_0 ✗ Type II Error $p = \beta$
Reject H_0 ✗ Type I Error $p = \alpha$	Reject H_0 ✓ Statistical Power $p = 1 - \beta$

Decisions

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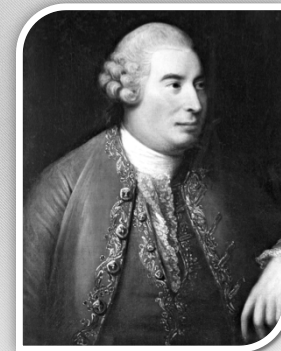


We NEVER “accept” or “prove” the Null Hypothesis on the basis of sample data

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Sir Francis Bacon
1561 - 1626



David Hume
1711 - 1776

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A lack of evidence for an effect in a sample is not good evidence for the lack of an effect in the population

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“Absence of evidence is not evidence of absence”


Altman and Bland (1995)

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Truth	
There is No Effect H_0 is True	There is an Effect H_1 is True
Fail to Reject H_0 Specificity $p = 1 - \alpha$	Fail to Reject H_0 Type II Error $p = \beta$
Reject H_0 Type I Error $p = \alpha$	Reject H_0 Statistical Power $p = 1 - \beta$

Truth	
There is No Effect H_0 is True	There is an Effect H_1 is True
Fail to Reject H_0 Specificity $p = 1 - \alpha$	Fail to Reject H_0 Type II Error $p = \beta$
Reject H_0 Type I Error $p = \alpha$	Reject H_0 Statistical Power $p = 1 - \beta$

Statistical Power

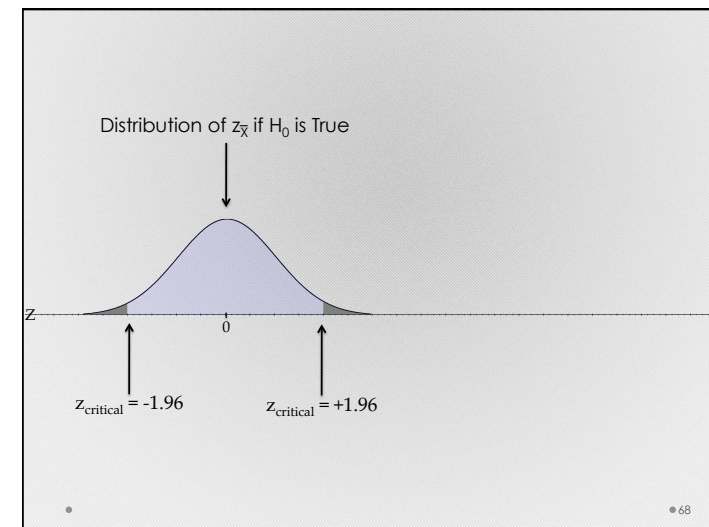
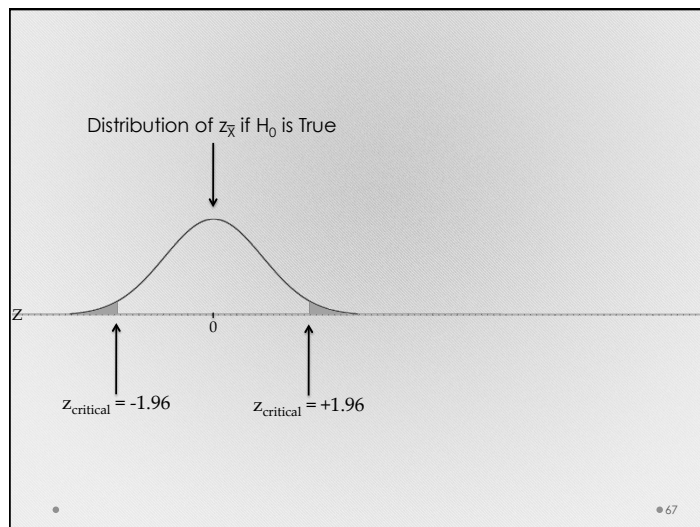
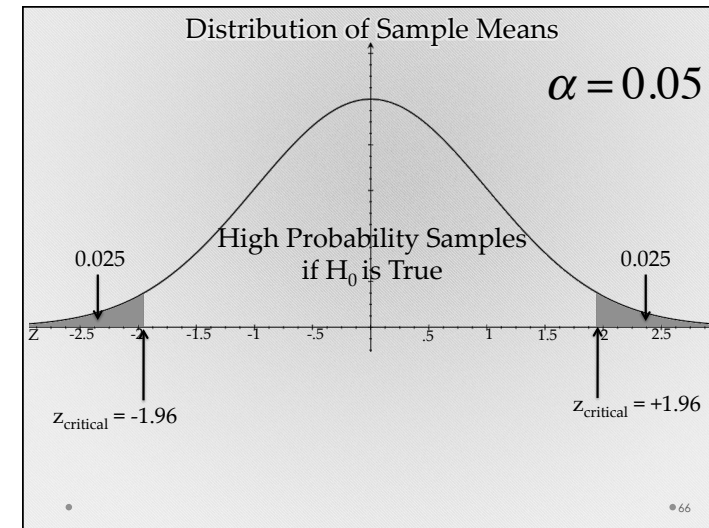


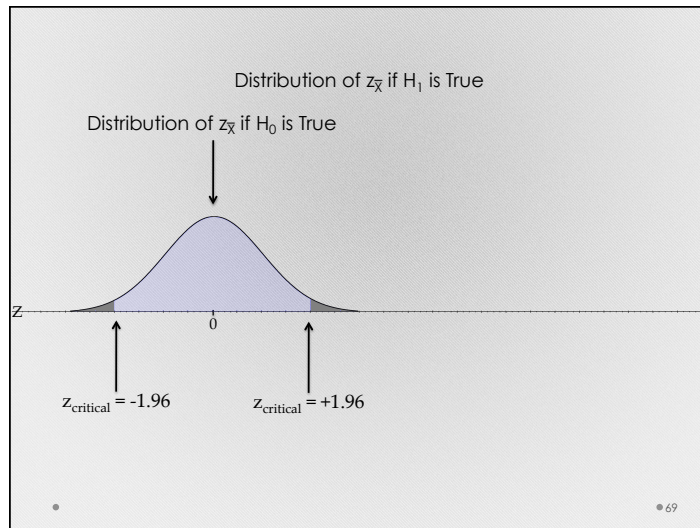
- Visualizing Statistical Power
- Factors Affecting Statistical Power
- Calculating Power*

Statistical Power

The power of a test is the probability that the test will reject a false null hypothesis.
 Power is the probability of *detecting a real effect*.

		Truth	
		There is No Effect H_0 is True	There is an Effect H_1 is True
Decisions	Fail to Reject H_0	<div> <div>✓</div> <div>Specificity $p = 1 - \alpha$</div> </div>	<div> <div>✗</div> <div>Type II Error $p = \beta$</div> </div>
	Reject H_0	<div> <div>✗</div> <div>Type I Error $p = \alpha$</div> </div>	<div> <div>✓</div> <div>Statistical Power $p = 1 - \beta$</div> </div>





Statistical Hypotheses

Null Hypothesis (H_0)

$\mu_{\text{treatment}} = \mu_{\text{without treatment}}$

Alternative Hypothesis (H_1)

$\mu_{\text{treatment}} \neq \mu_{\text{without treatment}}$

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Statistical Hypotheses

Null Hypothesis (H_0)

$\mu_{\text{treatment}} = \mu_{\text{without treatment}}$

Alternative Hypothesis (H_1)

$\mu_{\text{treatment}} \neq \mu_{\text{without treatment}}$

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Statistical Hypotheses

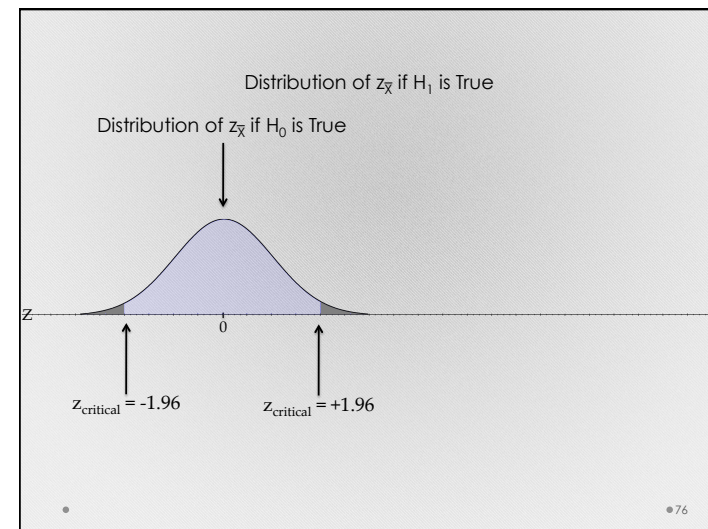
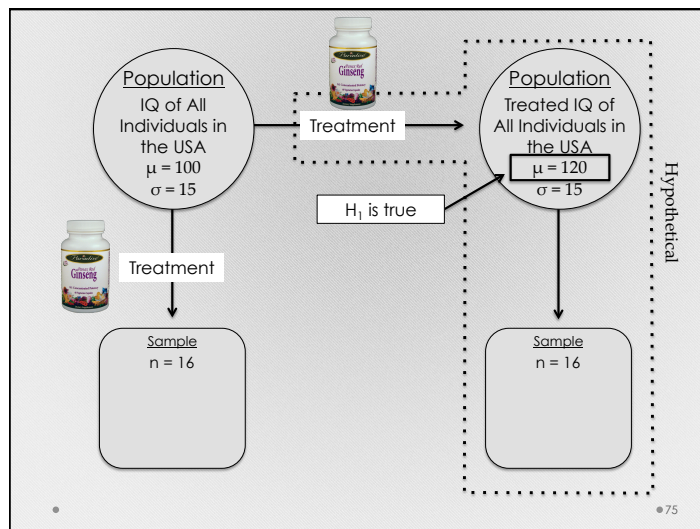
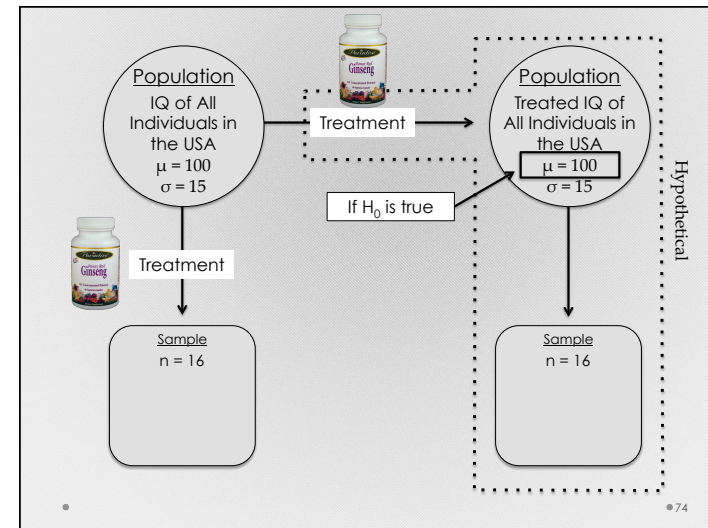
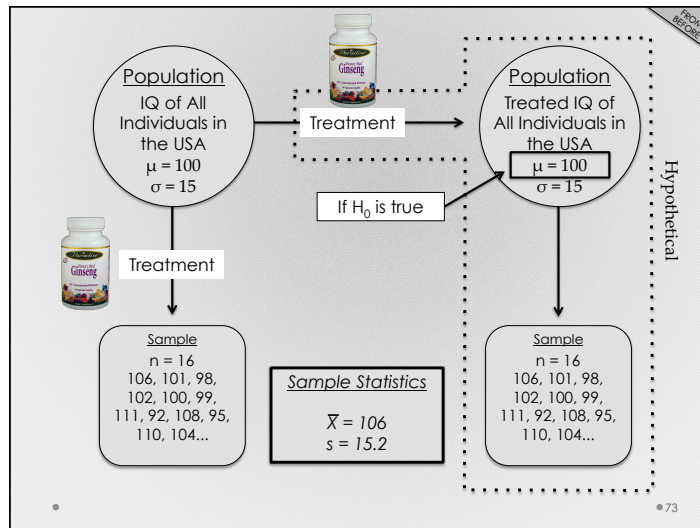
Null Hypothesis (H_0)

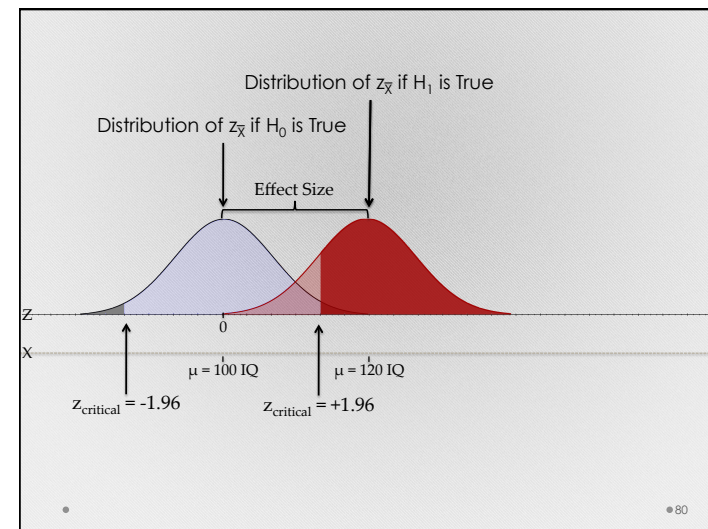
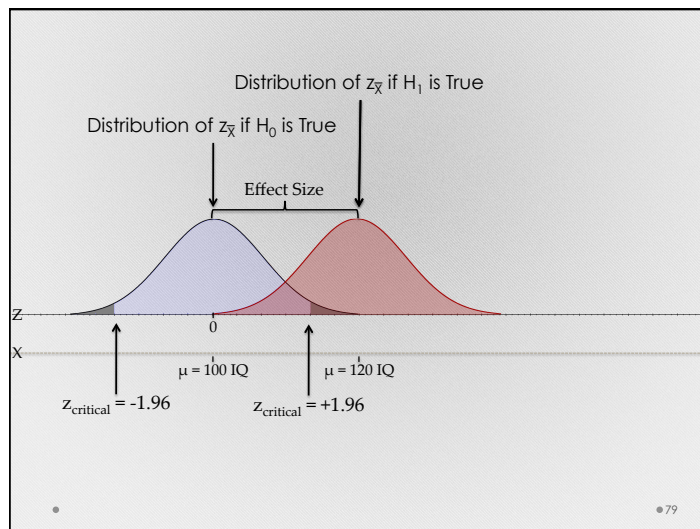
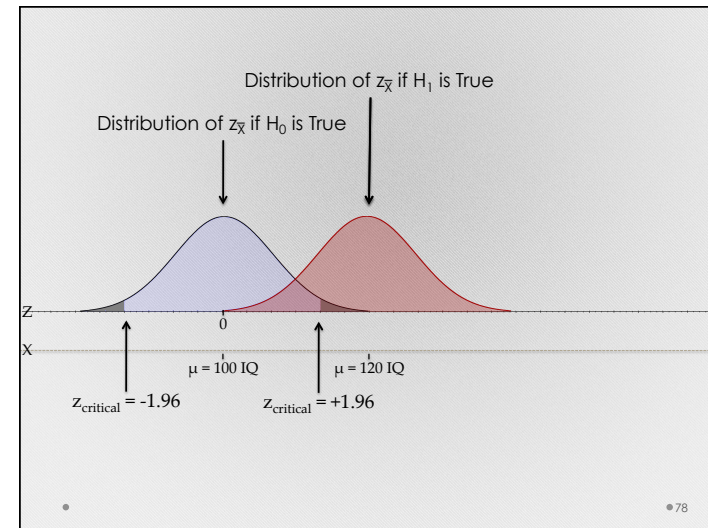
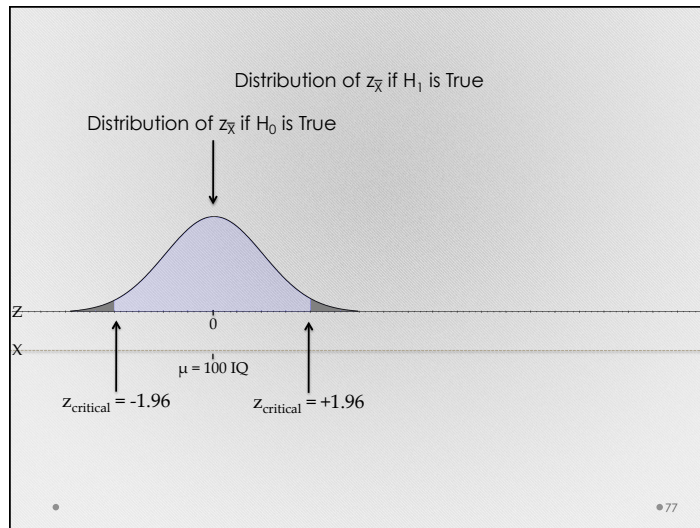
$\mu_{\text{treatment}} = \mu_{\text{without treatment}}$

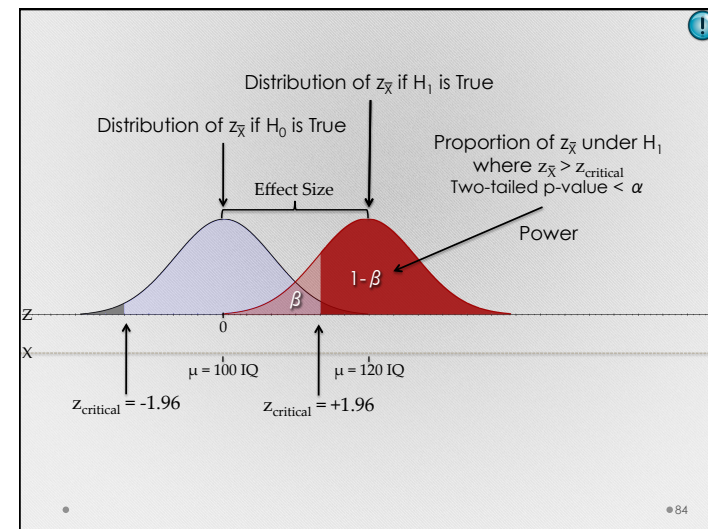
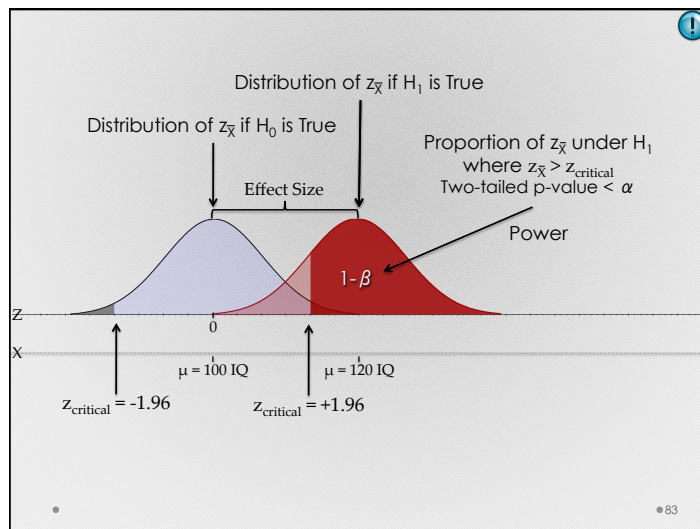
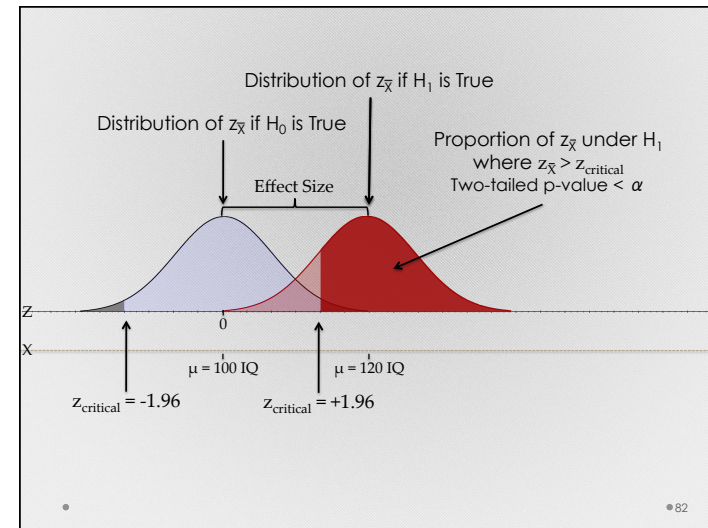
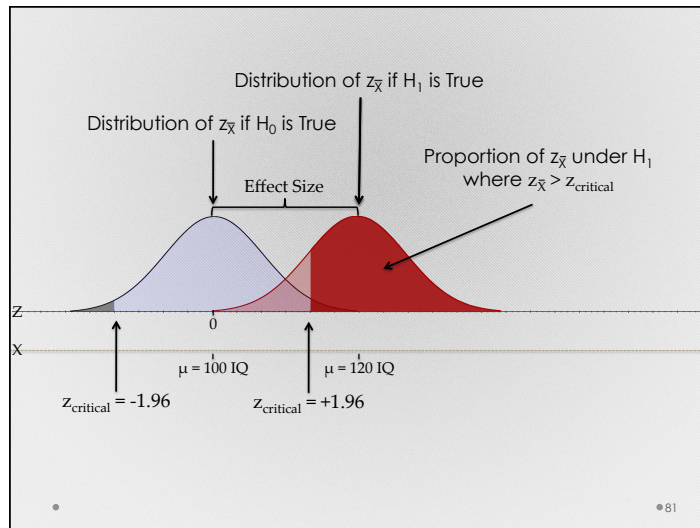
Alternative Hypothesis (H_1)

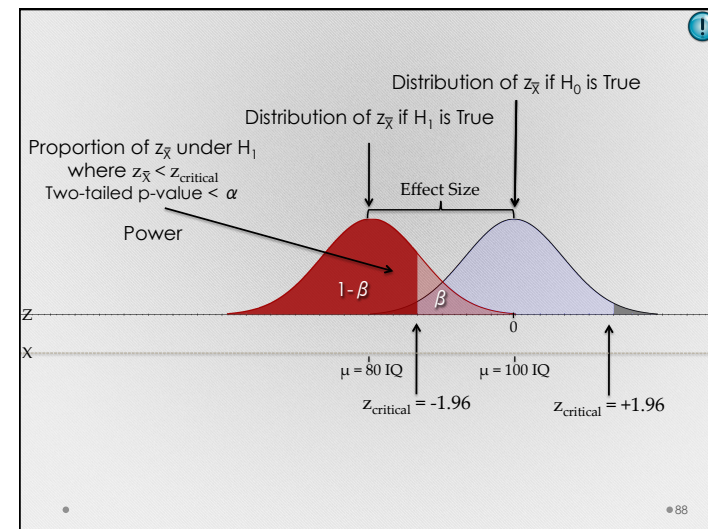
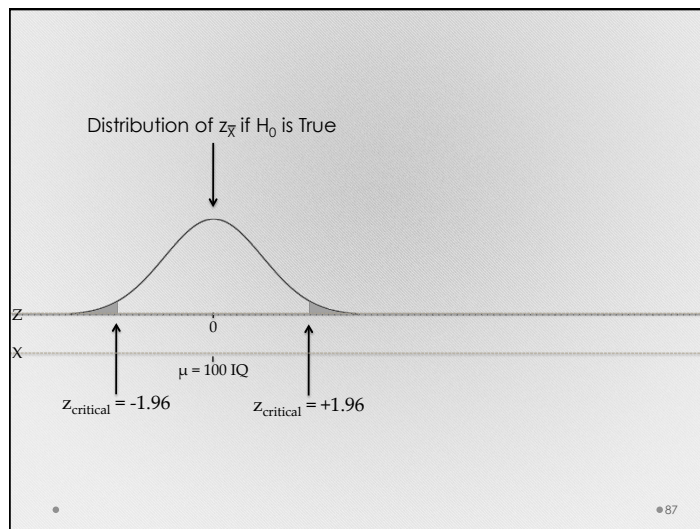
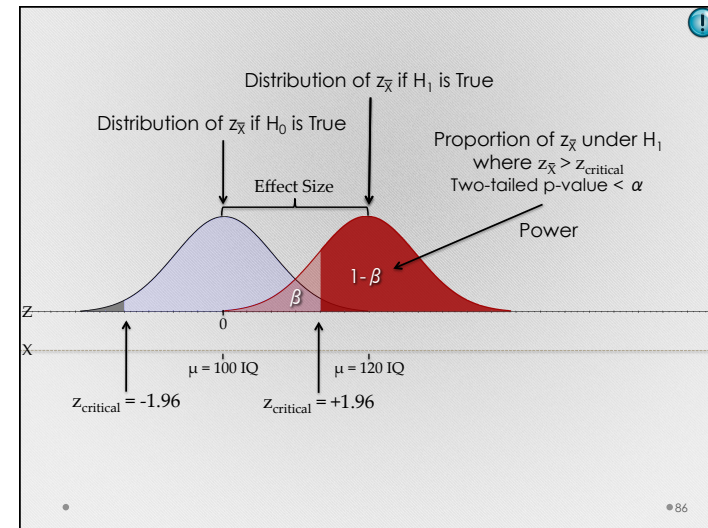
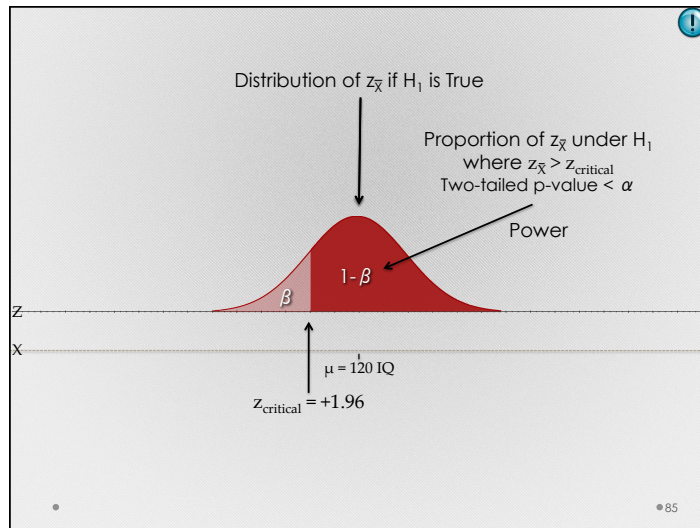
$\mu_{\text{treatment}} \neq \mu_{\text{without treatment}}$

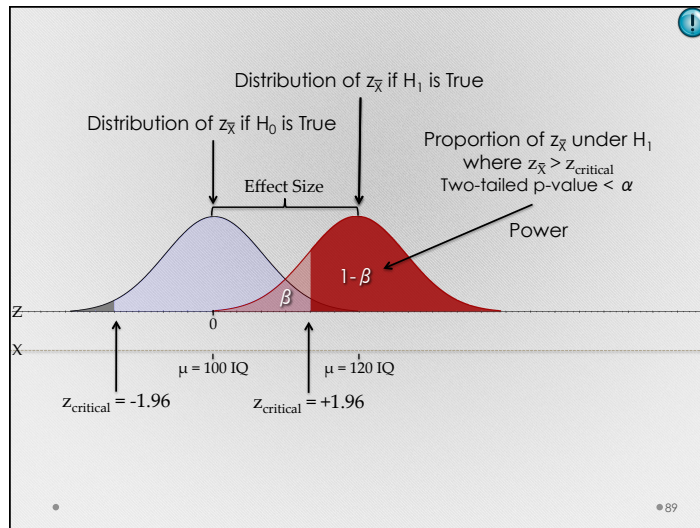
• 72











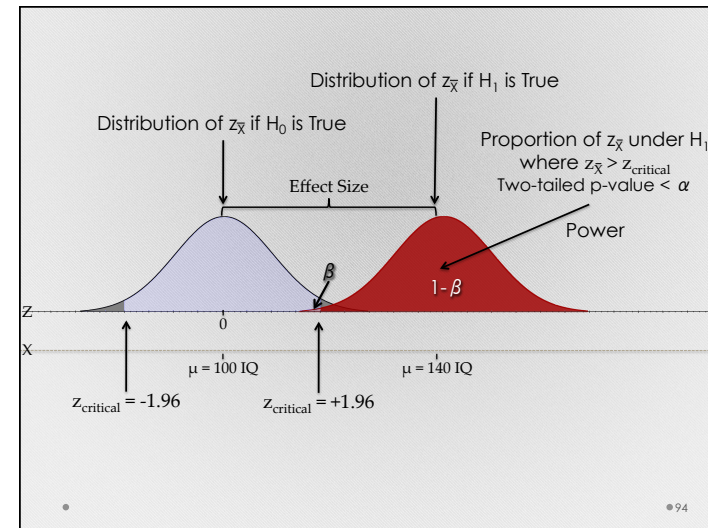
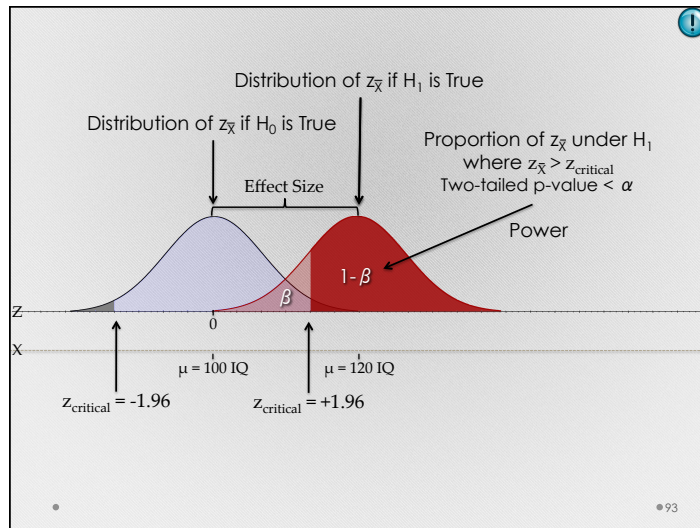
Factors That Affect Power

- Size of the effect in the population

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

Estimating $(\mu_{\text{alternative}} - \mu_{\text{null}})$

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$



Factors that Affect Power

- Size of the effect in the population
- Variability in the populations

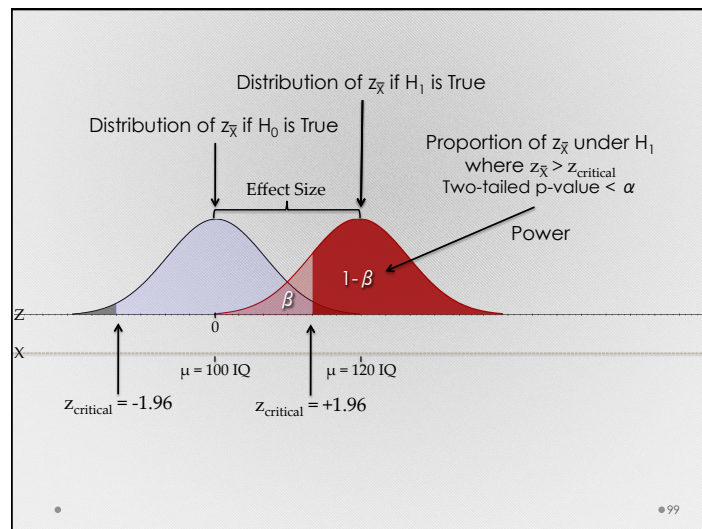
$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

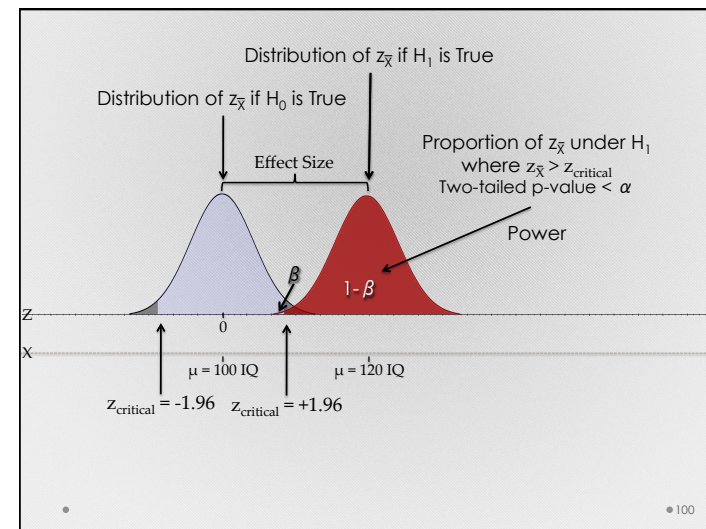
• 97

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

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Factors that Affect Power

- Size of the effect in the population
- Variability in the populations
- Sample Size

• 101

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

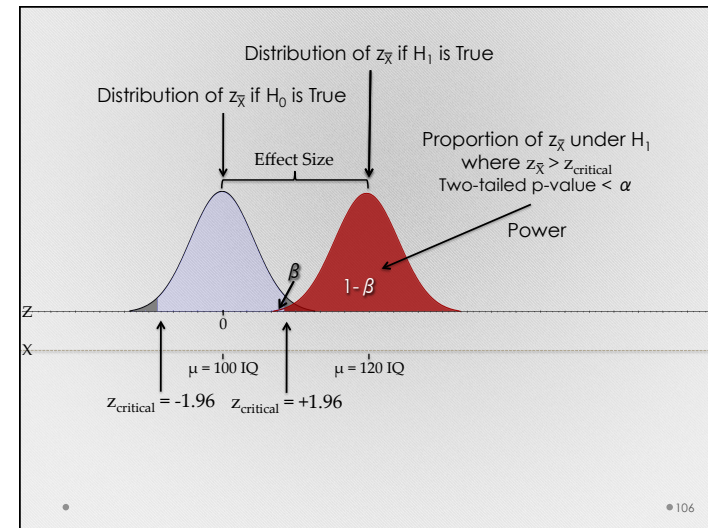
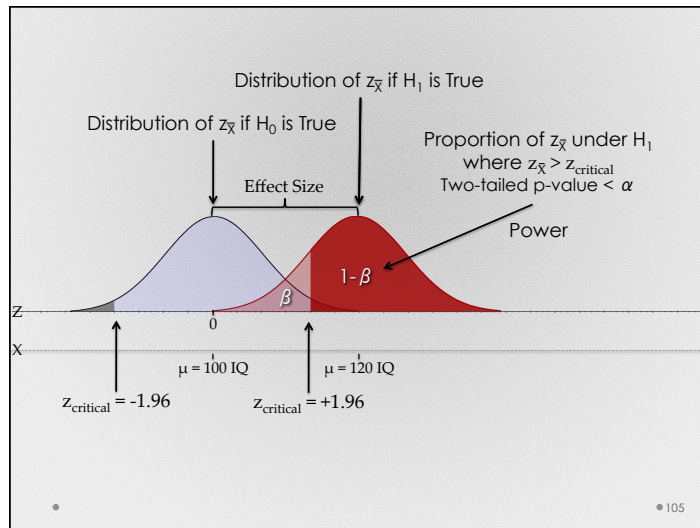
• 102

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

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$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

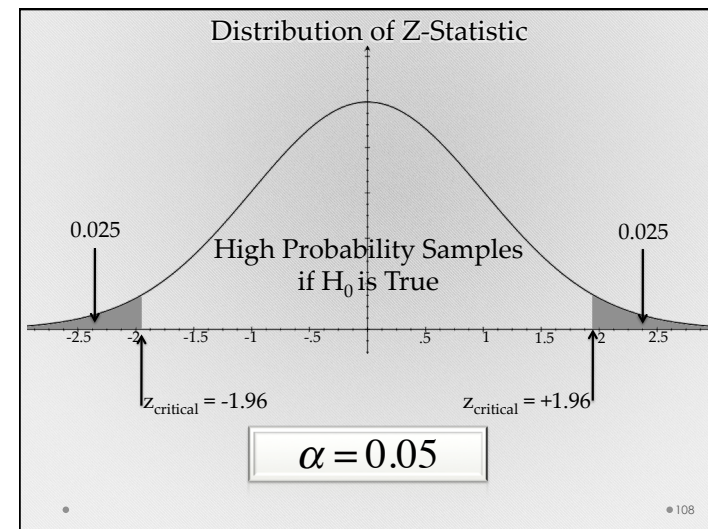
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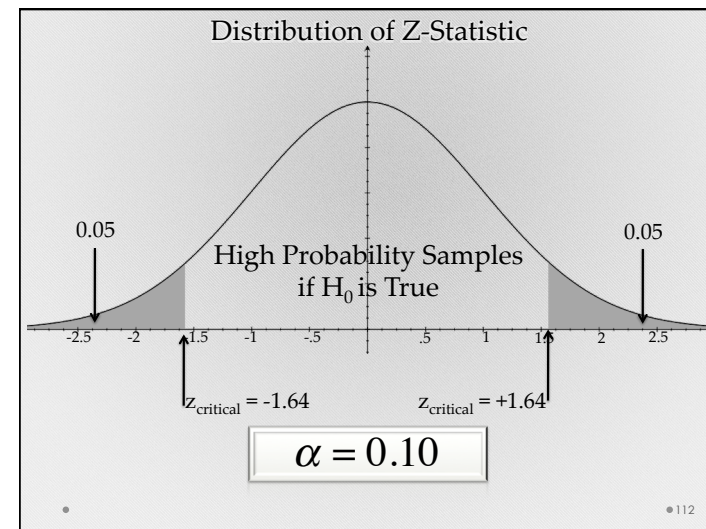
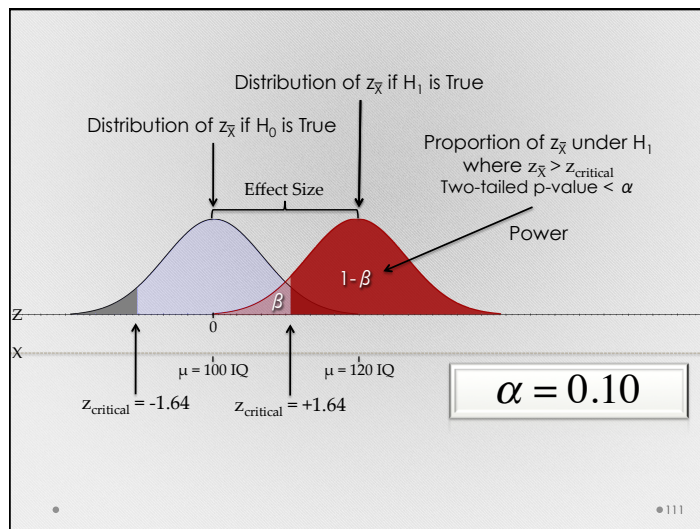
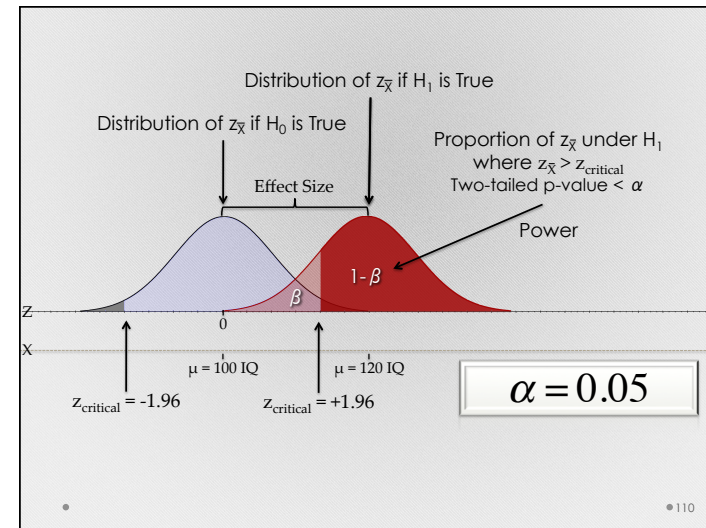
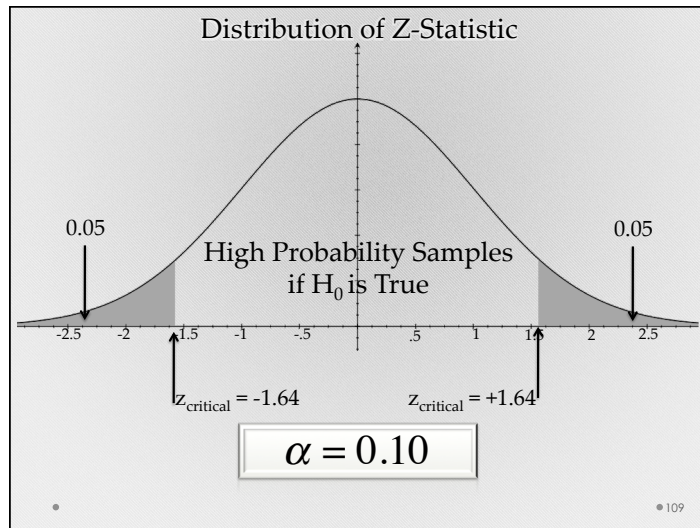


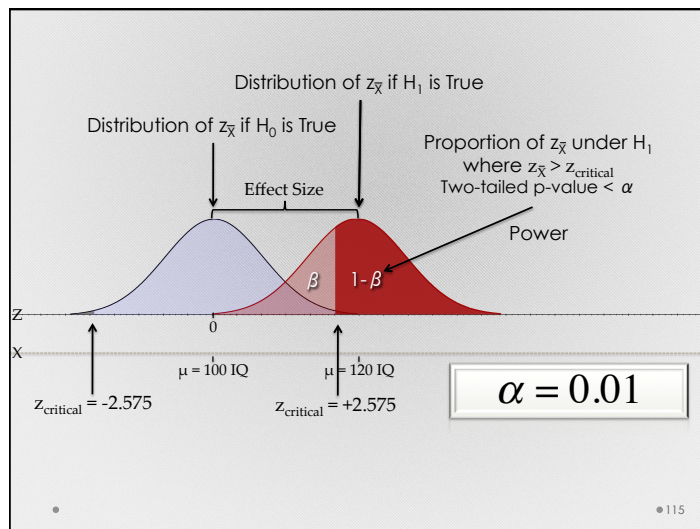
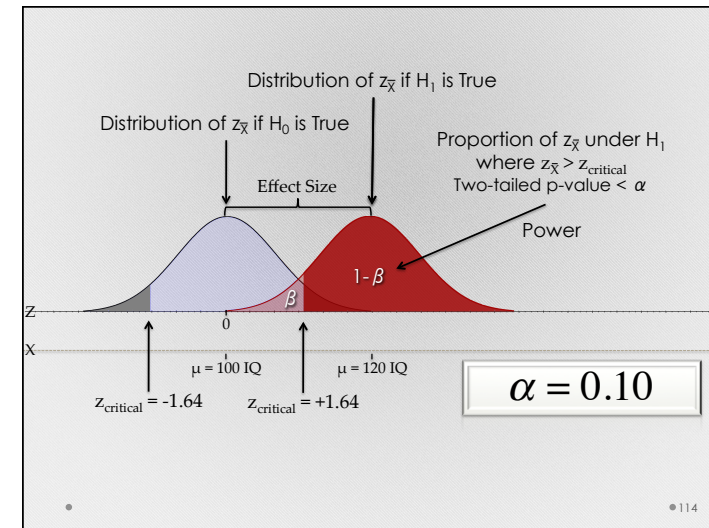
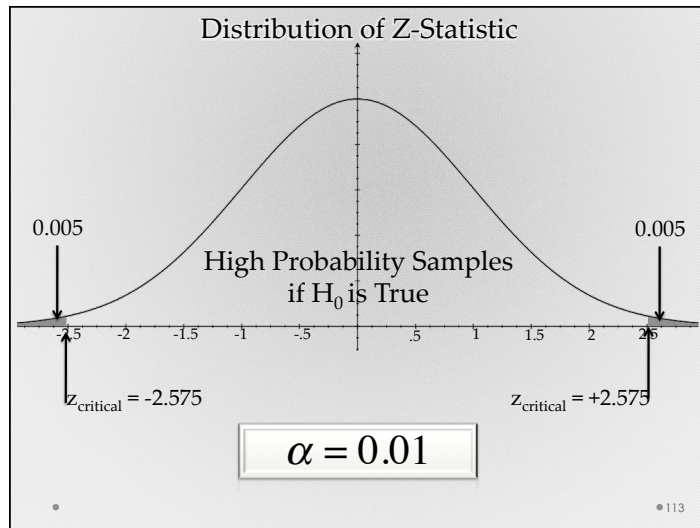
Factors that Affect Power

- Size of the effect in the population
- Variability in the populations
- Sample Size
- Alpha level

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Factors that Affect Power

- Size of the effect in the population
- Variability in the populations
- Sample Size
- Alpha level
- *Directional Hypotheses*

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Statistical Hypotheses

Null Hypothesis (H_0)

$$\mu_{\text{treatment}} = \mu_{\text{without treatment}}$$

Alternative Hypothesis (H_1)

$$\mu_{\text{treatment}} \neq \mu_{\text{without treatment}}$$

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Directional Statistical Hypotheses

Null Hypothesis (H_0)

$$\mu_{\text{treatment}} \leq \mu_{\text{without treatment}}$$

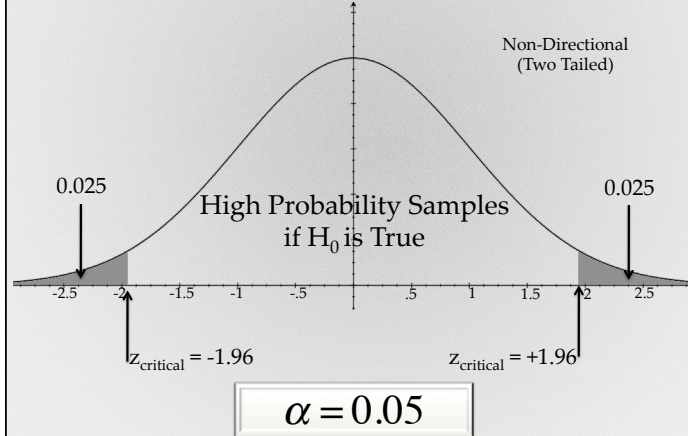
Alternative Hypothesis (H_1)

$$\mu_{\text{treatment}} > \mu_{\text{without treatment}}$$

Mutually Exclusive and Exhaustive

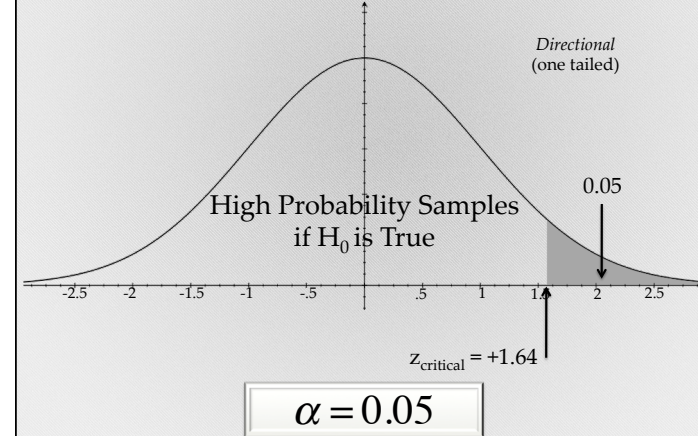
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Distribution of Z-Statistic

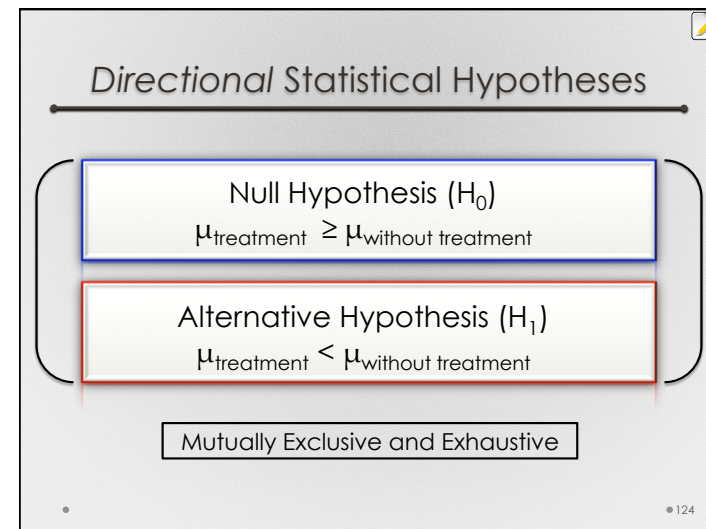
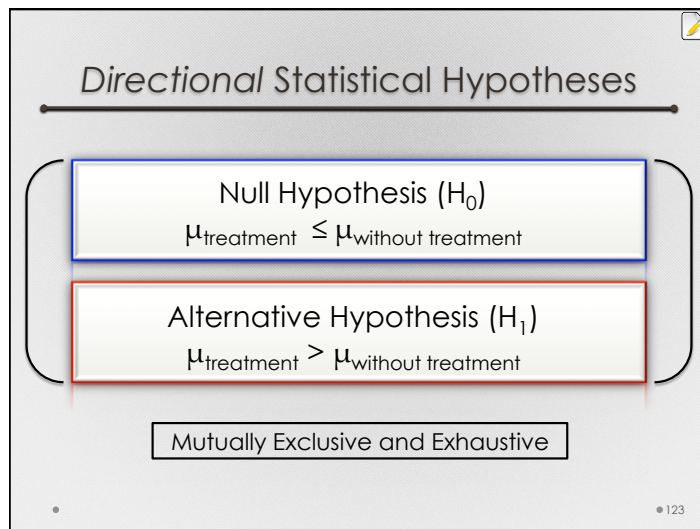
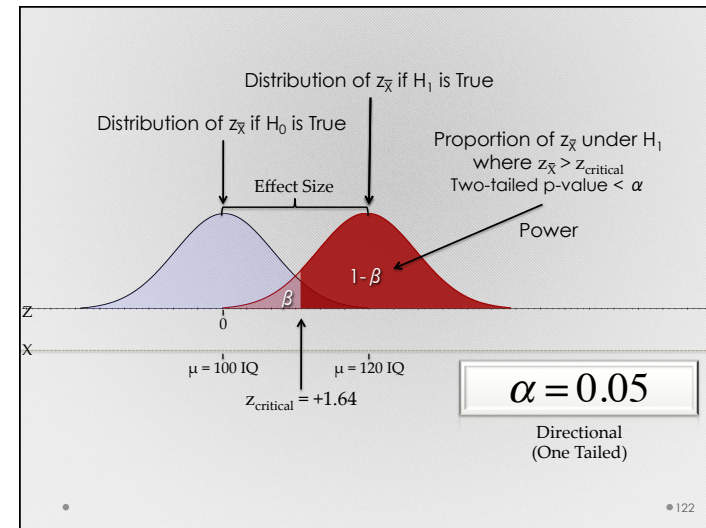
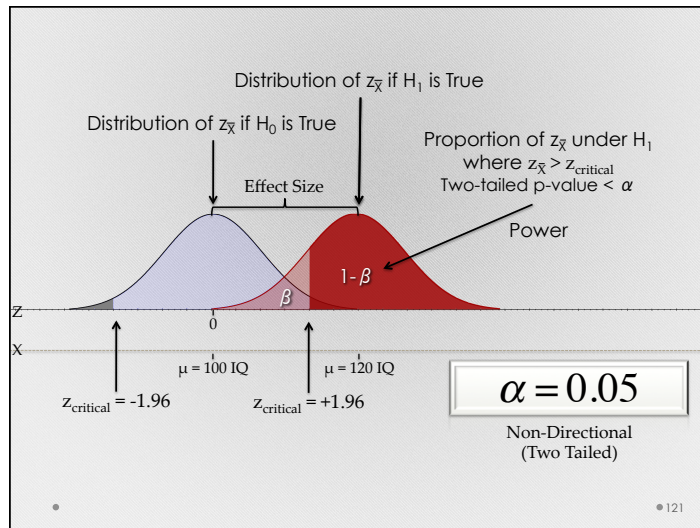


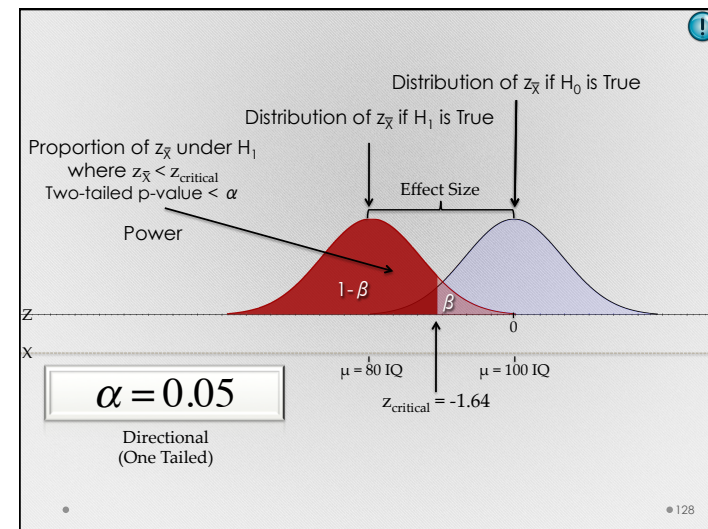
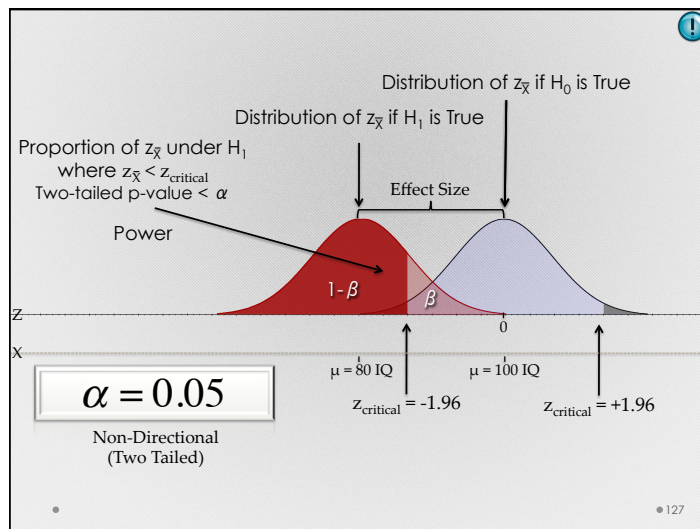
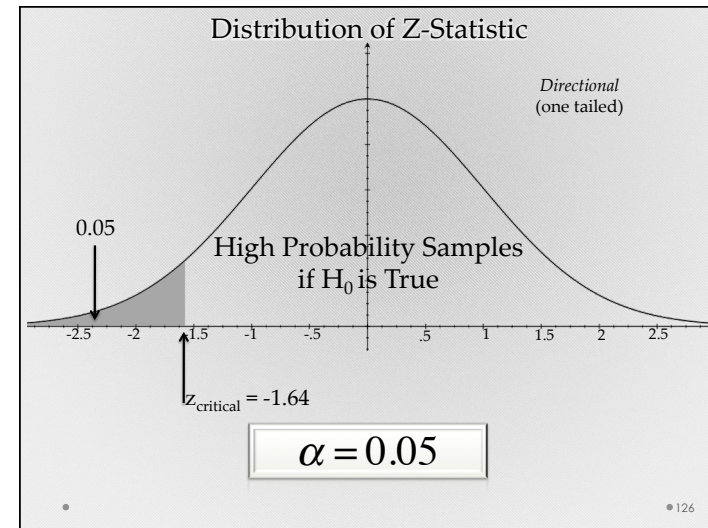
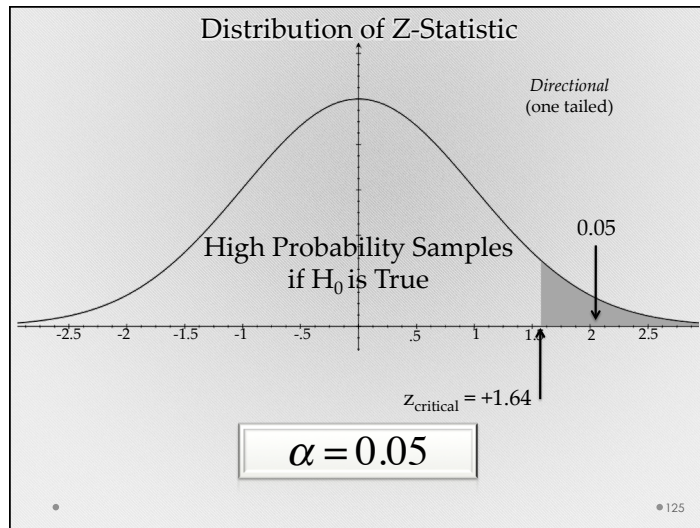
• 119

Distribution of Z-Statistic



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Directional Statistical Hypotheses

Null Hypothesis (H_0)

$\mu_{\text{treatment}} \geq \mu_{\text{without treatment}}$

Alternative Hypothesis (H_1)

$\mu_{\text{treatment}} < \mu_{\text{without treatment}}$

Mutually Exclusive and Exhaustive

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Directional Statistical Hypotheses

Null Hypothesis (H_0)

$\mu_{\text{treatment}} \leq \mu_{\text{without treatment}}$

Null Hypothesis (H_0)

$\mu_{\text{treatment}} \geq \mu_{\text{without treatment}}$

Alternative Hypothesis (H_1)

$\mu_{\text{treatment}} > \mu_{\text{without treatment}}$

Alternative Hypothesis (H_1)

$\mu_{\text{treatment}} < \mu_{\text{without treatment}}$

Mutually Exclusive and Exhaustive

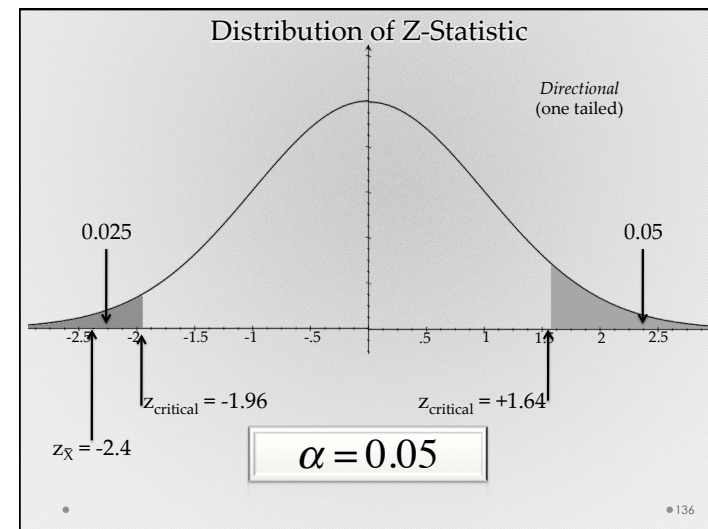
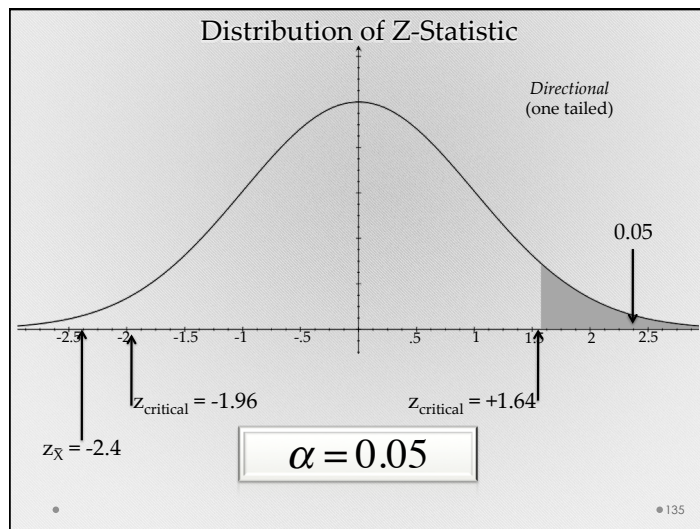
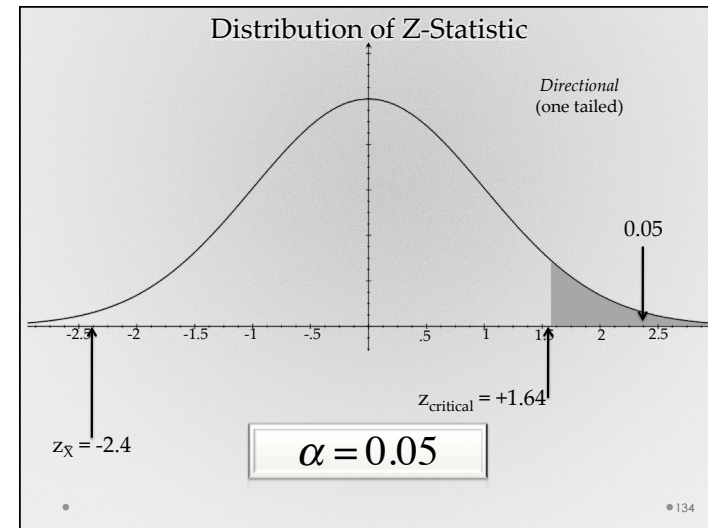
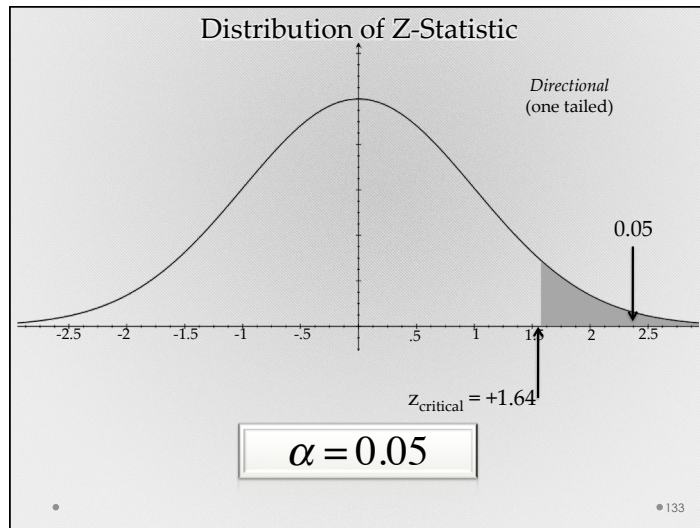
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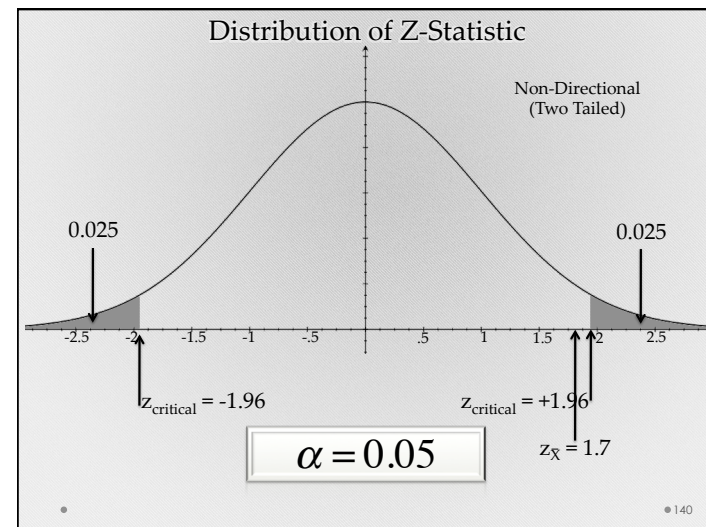
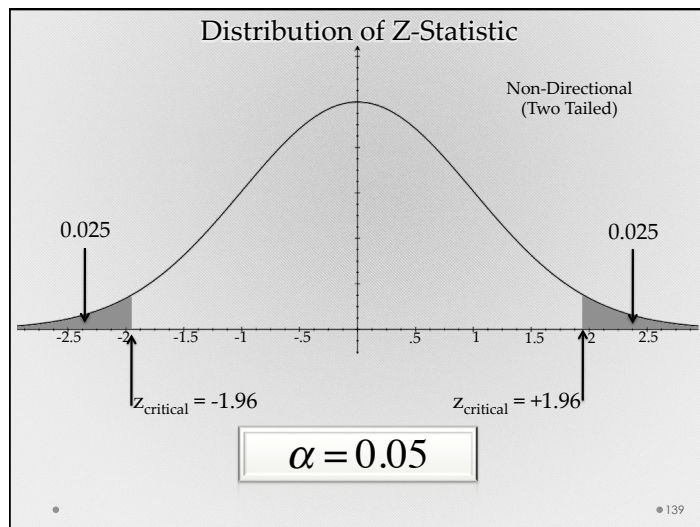
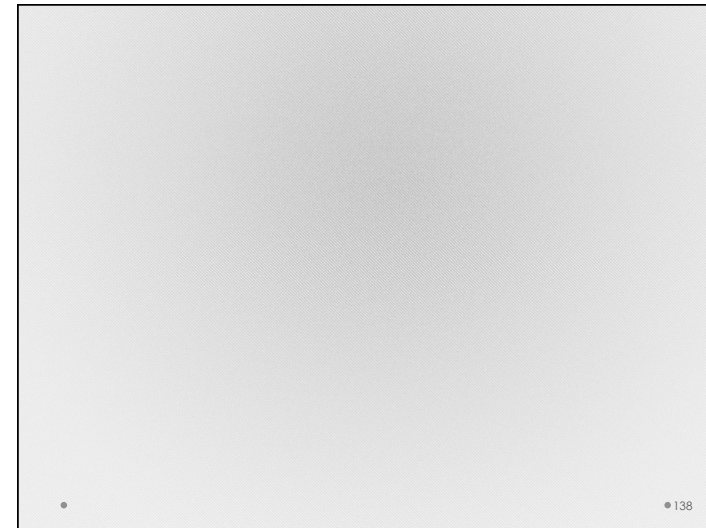
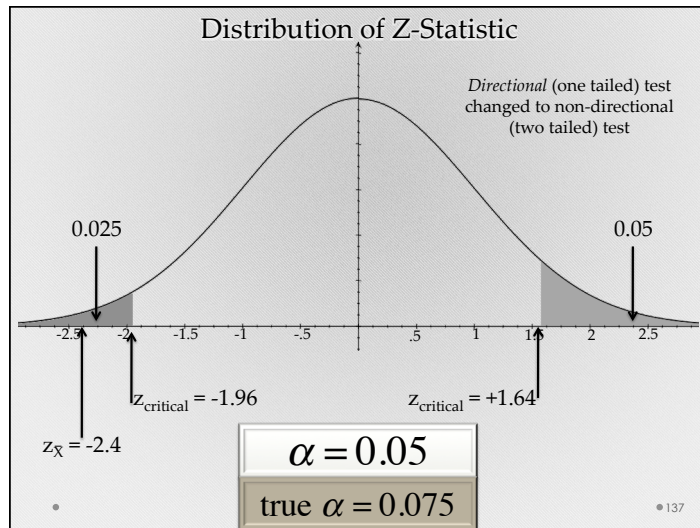
Directional Tests

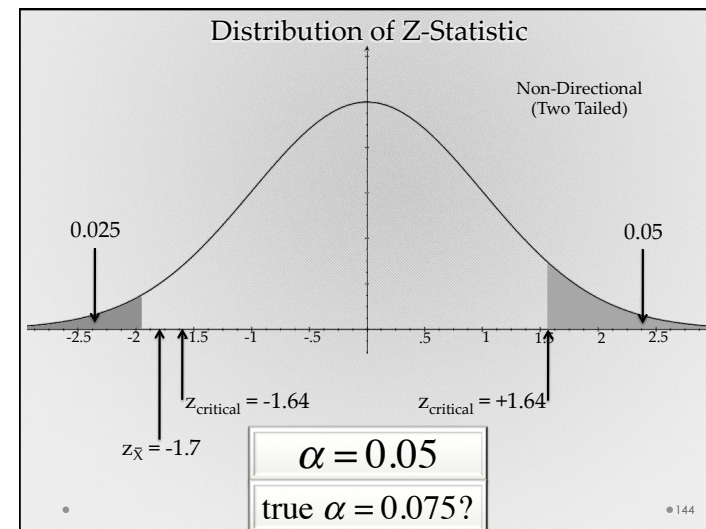
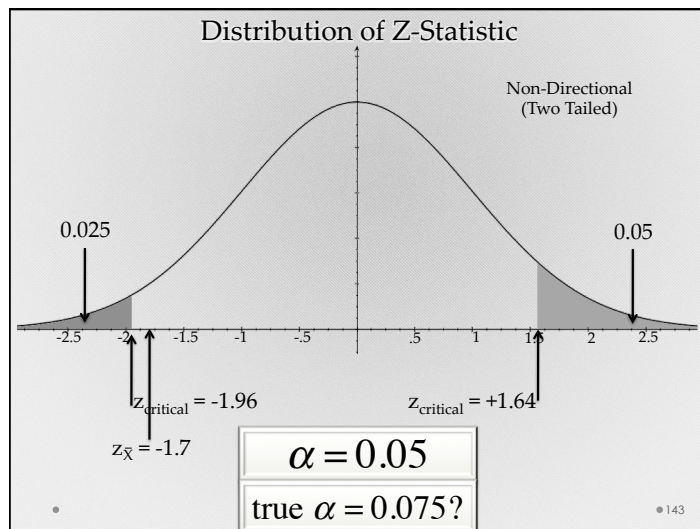
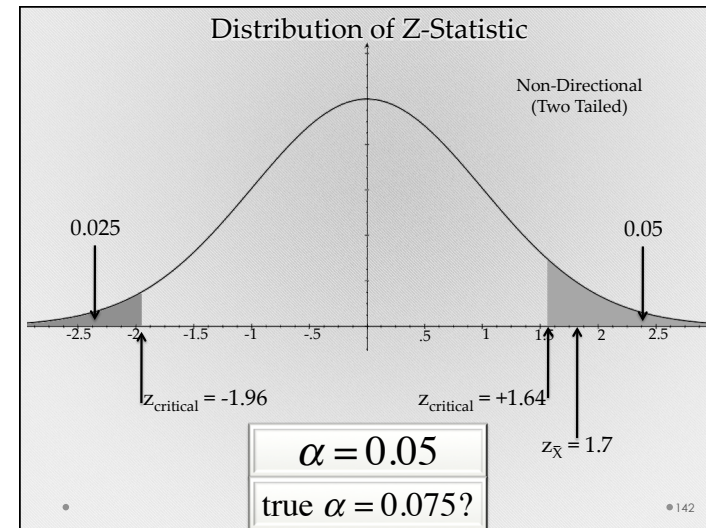
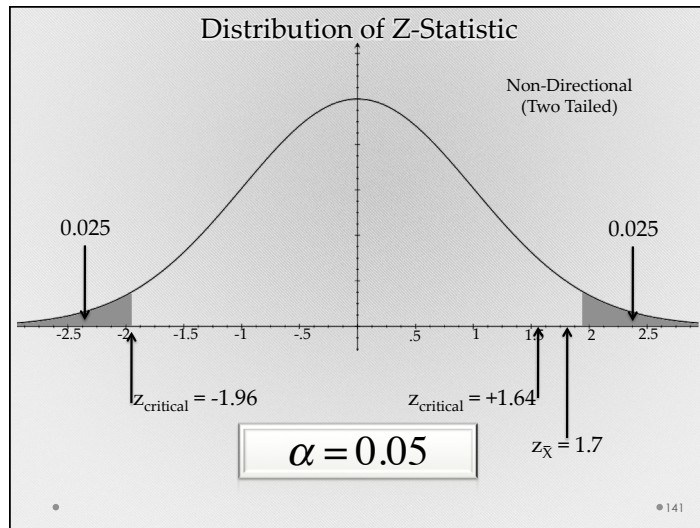
- Must specify before running study
- Effects in the other direction must be unimportant, uninteresting, or conceptually unlikely
- Potential for abuse makes them suspect to many researchers

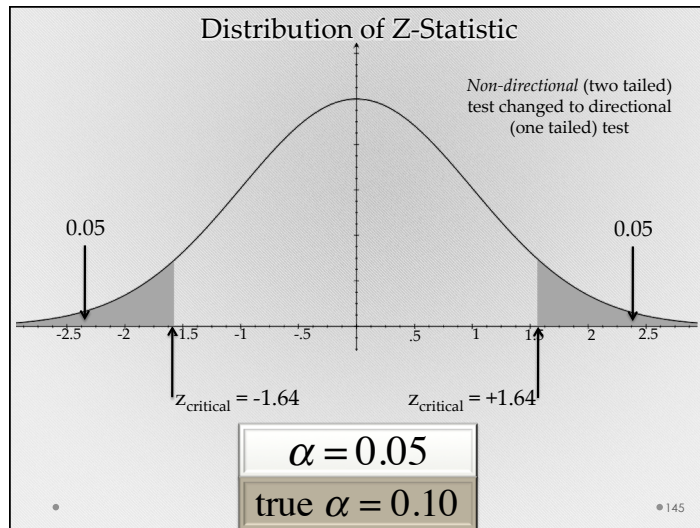
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Directional Tests

- Must specify before running study
- Effects in the other direction must be unimportant, uninteresting, or conceptually unlikely
- Potential for abuse makes them suspect to many researchers

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Factors that Affect Power

- Size of the effect in the population
- Variability in the populations
- Sample Size
- Alpha level
- Directional Hypotheses

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Check Your Understanding

Which of the following is guaranteed not to increase power?

- ☐ A • Decreasing the alpha level (e.g. 0.10 => 0.01)
- ☐ B • Increasing sample size
- ☐ C • Decreasing the population variability
- ☐ D • Increasing the size of the effect
- ☐ E • All of the above will increase power

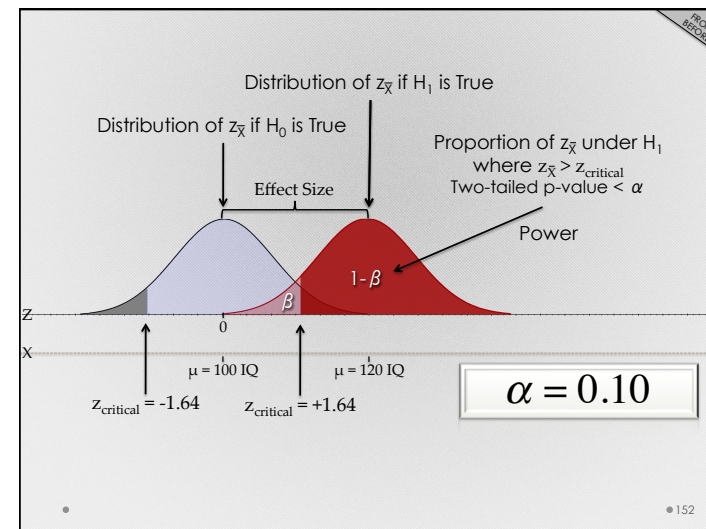
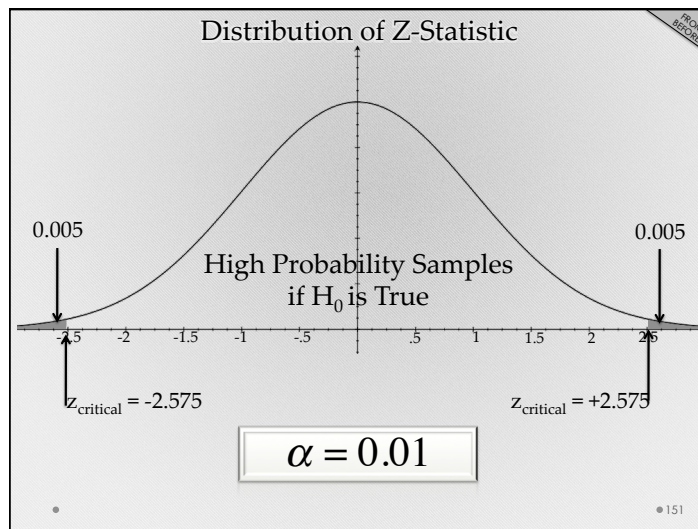
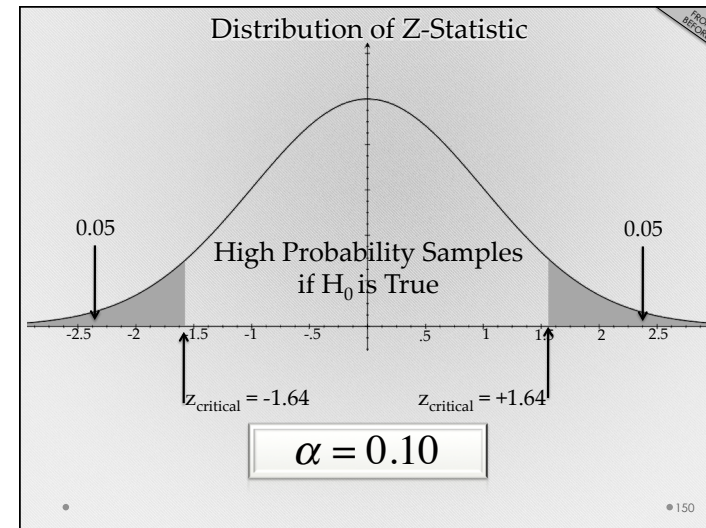
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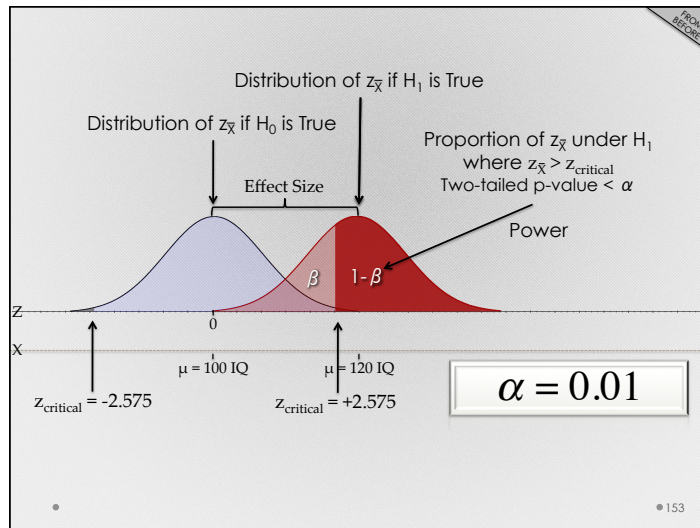
Check Your Understanding

Which of the following is guaranteed not to increase power?

- A** • Decreasing the alpha level (e.g. 0.10 \Rightarrow 0.01)
- B • Increasing sample size
- C • Decreasing the population variability
- D • Increasing the size of the effect
- E • All of the above will increase power

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Check Your Understanding

Which of the following is guaranteed not to increase power?

- ☒ A • Decreasing the alpha level (e.g. 0.10 \Rightarrow 0.01)
- ☐ B • Increasing sample size
- ☐ C • Decreasing the population variability
- ☐ D • Increasing the size of the effect
- ☐ E • All of the above will increase power

Factors that Affect Power

- Size of the effect in the population
- Variability in the populations
- Sample Size
- Alpha level
- *Directional Hypotheses*

Truth

There is No Effect
 H_0 is True

There is an Effect
 H_1 is True

Decisions

Fail to Reject H_0
✓ Specificity
 $p = 1 - \alpha$

Fail to Reject H_0
✗ Type II Error
 $p = \beta$

Reject H_0
✗ Type I Error
 $p = \alpha$

Reject H_0
✓ Statistical Power
 $p = 1 - \beta$

Statistical Errors	
<p>Type I Error Alpha Error "False Alarm"</p> <p><u>Factors Affecting Error Rate</u> Alpha Level</p>	<p>Type II Error Beta Error "Miss"</p> <p><u>Factors Affecting Error Rate</u> Alpha Level Effect Size Population Variance Sample Size Directional Hypotheses</p>