


The Distribution  
of Sample Means



The Logic of  
Hypothesis Testing

● 1

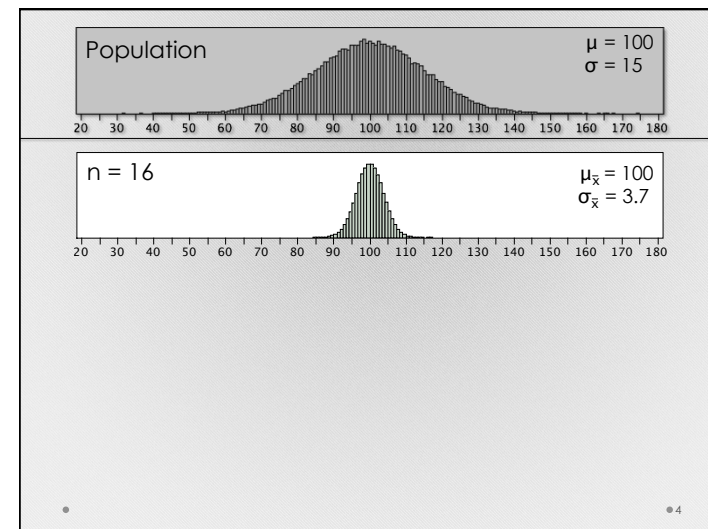
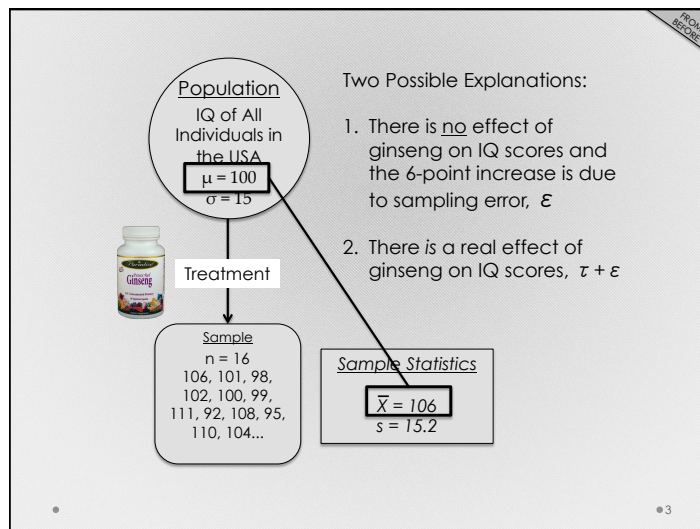


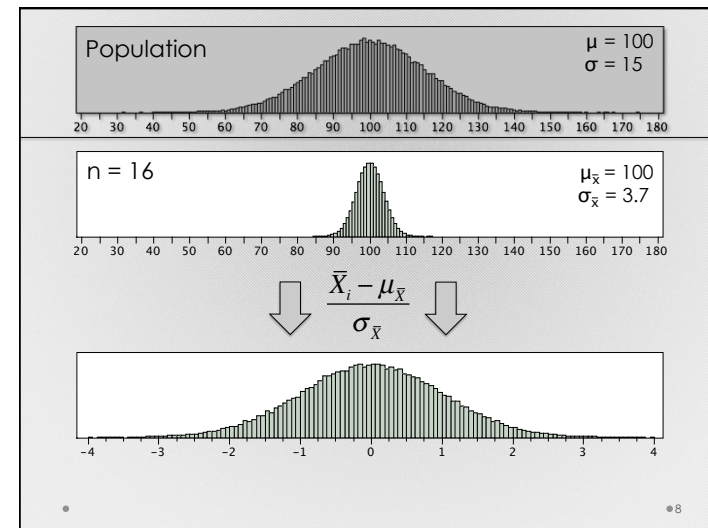
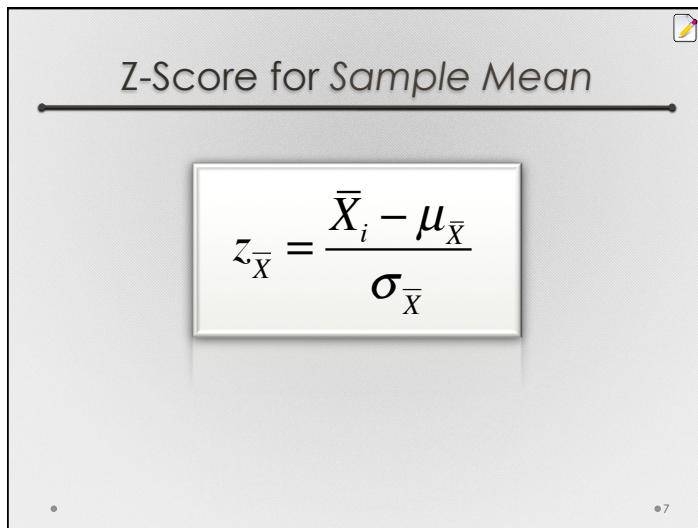
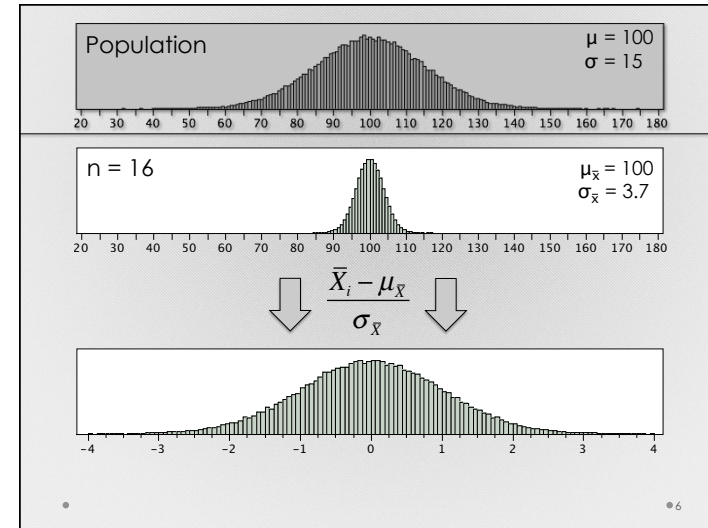
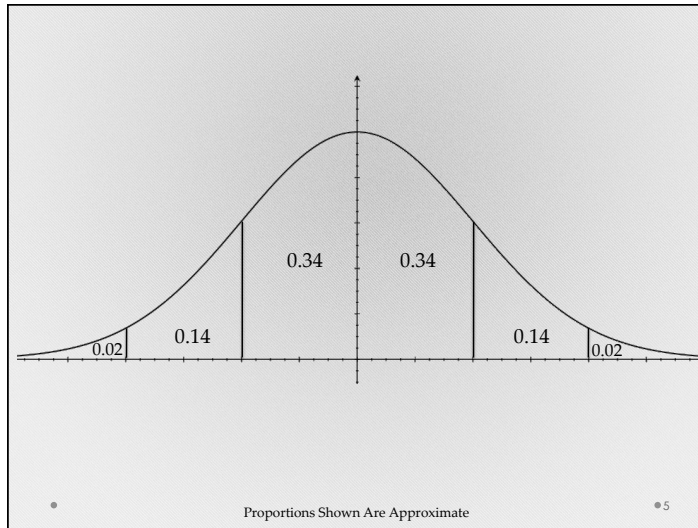
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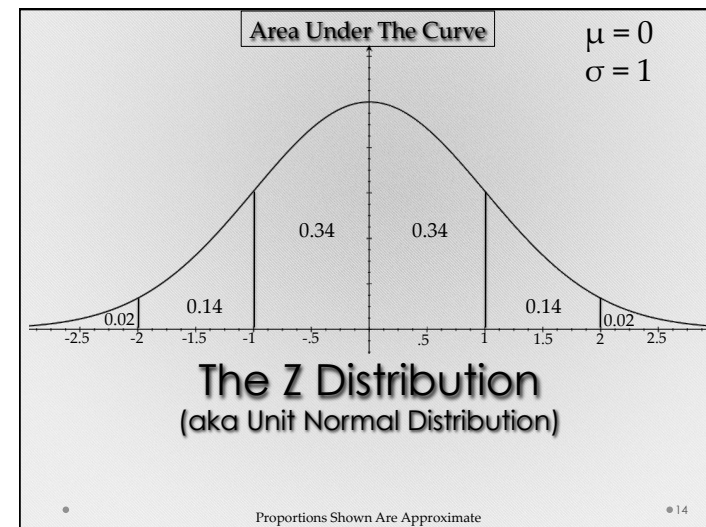
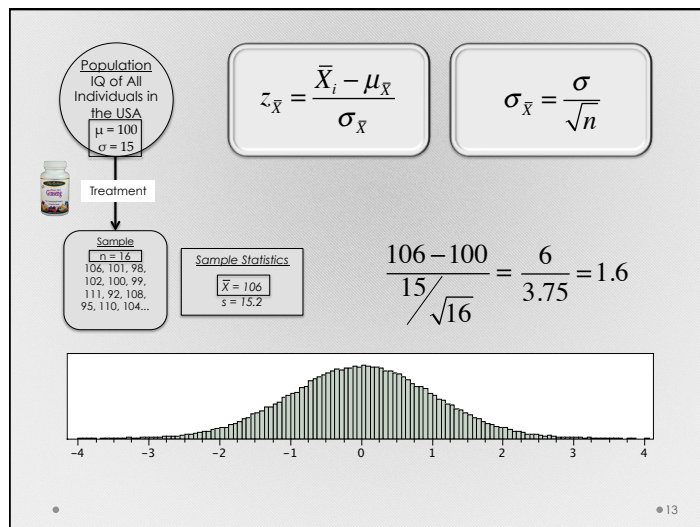
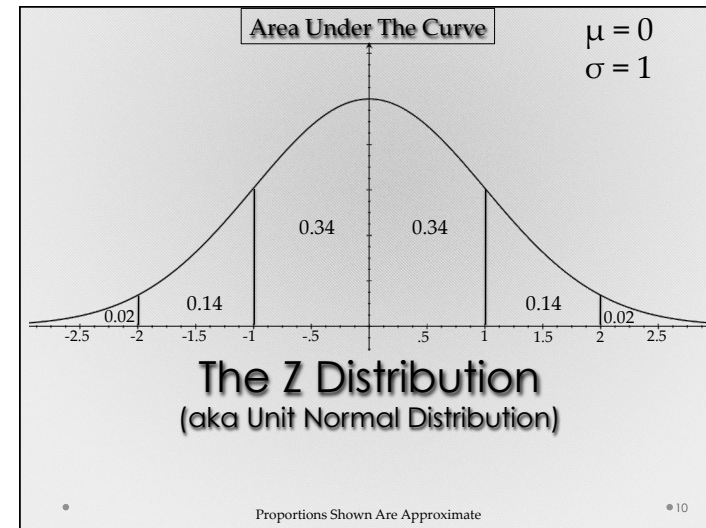
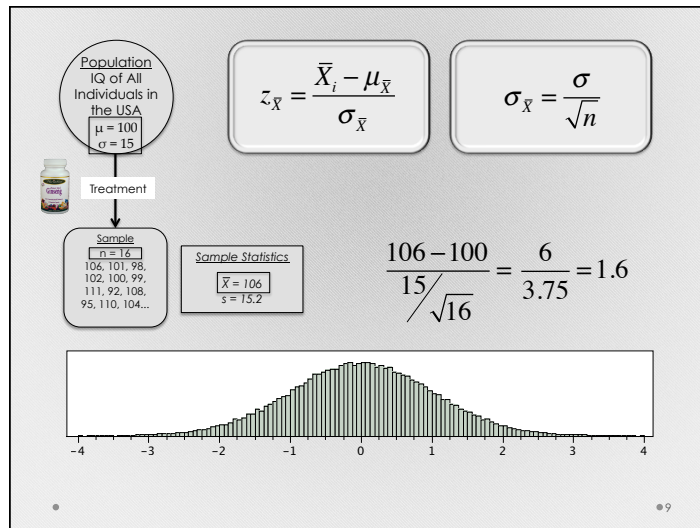


The Logic of  
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● 2







## Proportion More Extreme Value

a.k.a. p-value

A p-value is way of describing how *extreme* a score is in a distribution. A p-value is the proportion of a distribution more extreme than a given score.

One-Tailed P-Value:

Two-Tailed P-Value:

Proportion of distribution more extreme in one tail

Proportion of distribution more extreme in both tails

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Population  
IQ of All Individuals in the USA  
 $\mu = 100$   
 $\sigma = 15$

Treatment

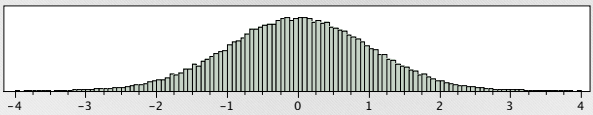
Sample  
 $n = 16$   
106, 101, 98, 102, 100, 99, 111, 92, 108, 95, 110, 104...

Sample Statistics  
 $\bar{X} = 106$   
 $s = 15.2$

$$z_{\bar{X}} = \frac{\bar{X}_i - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

$$\frac{106 - 100}{15/\sqrt{16}} = \frac{6}{3.75} = 1.6$$



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Distribution and Probability Calculator v2.037

**Distribution Characteristics**

Distribution: Normal

Parameters

Mean: 0

Std. Dev.: 1

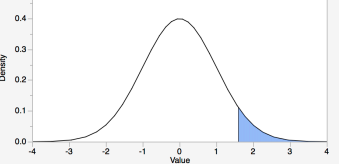
Reset

**Type of Calculation**

☒ Input Values and Calculate Probability

☐ Input Probability and Calculate Values

**Distribution**



**Calculations**

Probability Options

☐ Less than Value

☒ Greater than Value

☐ Between Value 1 and Value 2

☐ Outside Value 1 and Value 2

Input Value: 1.6

Probability = 0.0548

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Distribution and Probability Calculator v2.037

**Distribution Characteristics**

Distribution: Normal

Parameters

Mean: 0

Std. Dev.: 1

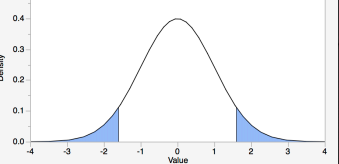
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**Type of Calculation**

☒ Input Values and Calculate Probability

☐ Input Probability and Calculate Values

**Distribution**



**Calculations**

Probability Options

☐ Less than Value

☐ Greater than Value

☐ Between Value 1 and Value 2

☒ Outside Value 1 and Value 2

Input Value 1: -1.6

Value 2: 1.6

Probability = 0.1096

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The Distribution  
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## The Logic of Hypothesis Testing

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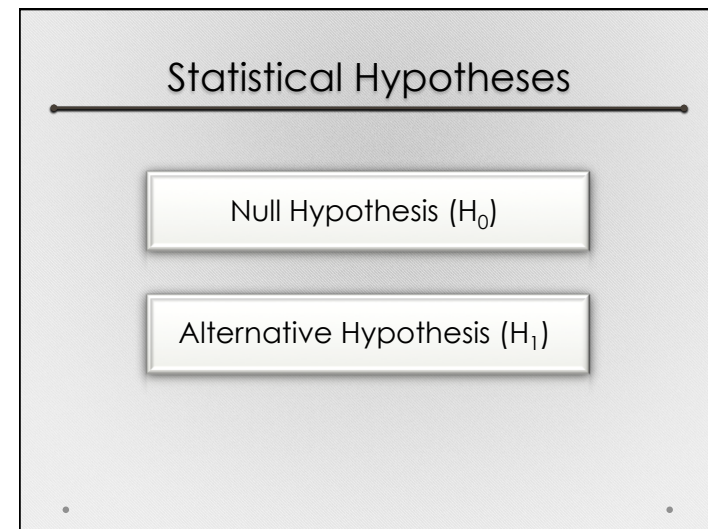
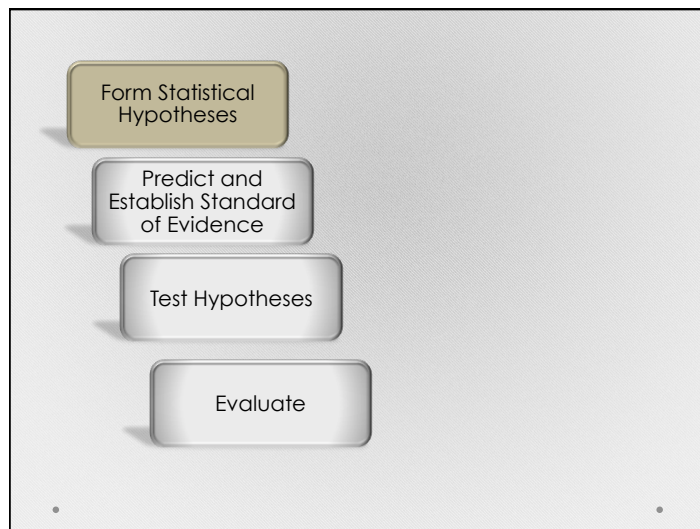
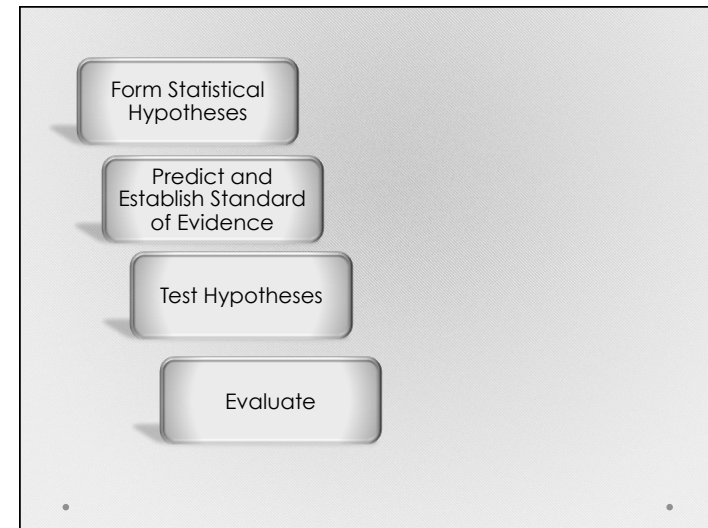
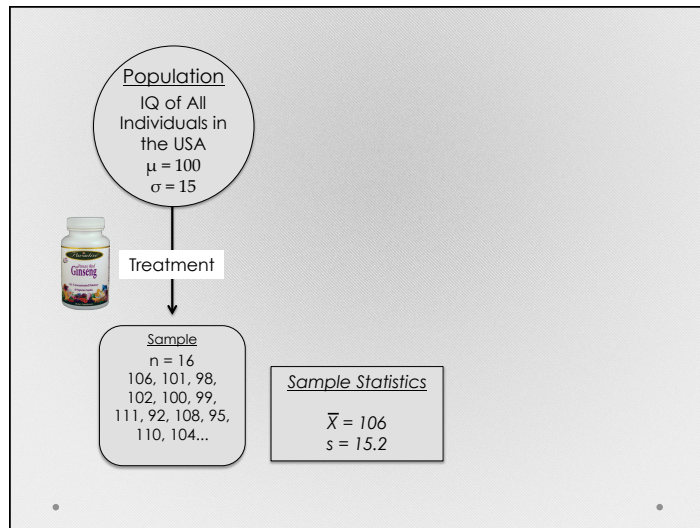
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## Hypothesis Test

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A hypothesis test is a statistical method that uses sample data to evaluate a hypothesis about a population

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## Null Hypothesis

( $H_0$ )

In the population there is no change, no difference, or no relationship. For an experiment, this means *no effect of treatment*

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**The difference observed  
is due to sampling error only**

## Alternative Hypothesis

( $H_1$ )

In the population there is a change, a difference, or a relationship. For an experiment, this means *an effect of treatment*

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( $H_1$ )

In the population there is a change, a difference, or a relationship. For an experiment, this means *an effect of treatment*

**The difference observed  
is due to sampling error  
AND a real effect**

### Statistical Hypotheses

Null Hypothesis ( $H_0$ )  
the treatment does not have an effect

Alternative Hypothesis ( $H_1$ )  
the treatment *has* an effect

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Mutually Exclusive and Exhaustive

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### Statistical Hypotheses

Null Hypothesis ( $H_0$ )  
 $\mu_{\text{treatment}} = \mu_{\text{without treatment}}$

Alternative Hypothesis ( $H_1$ )  
the treatment *has* an effect



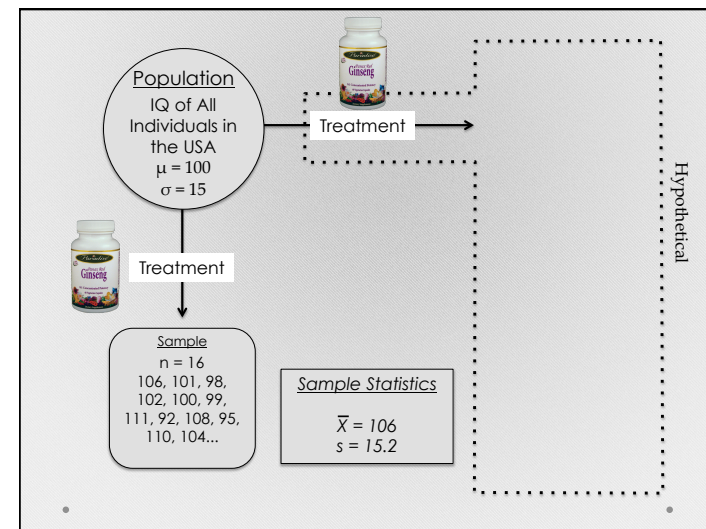
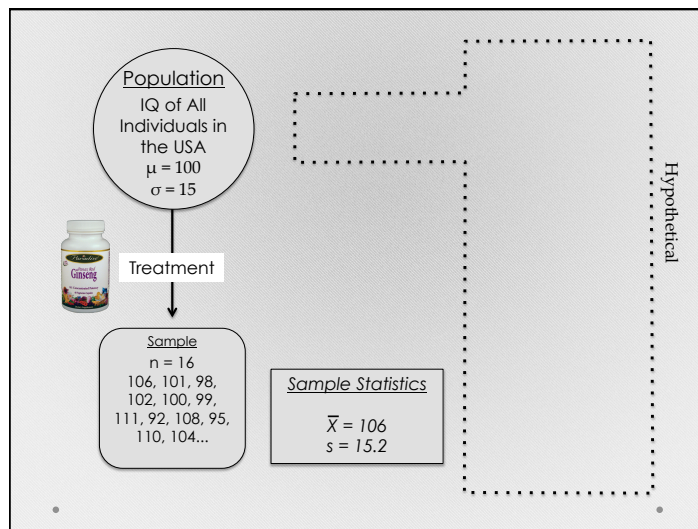
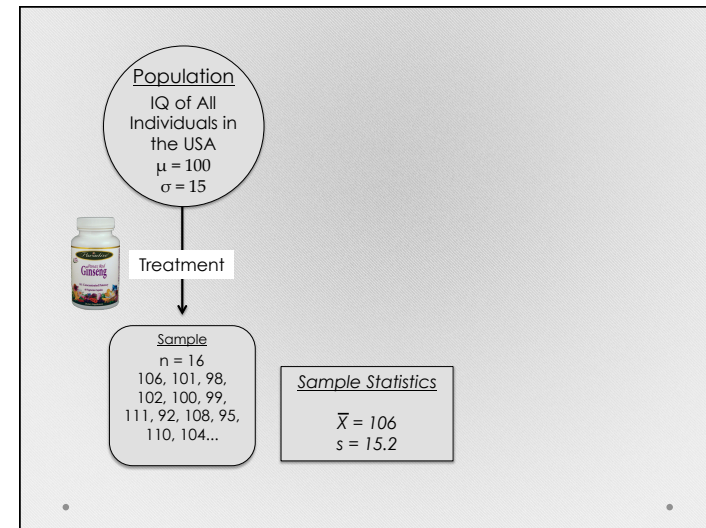
## Statistical Hypotheses

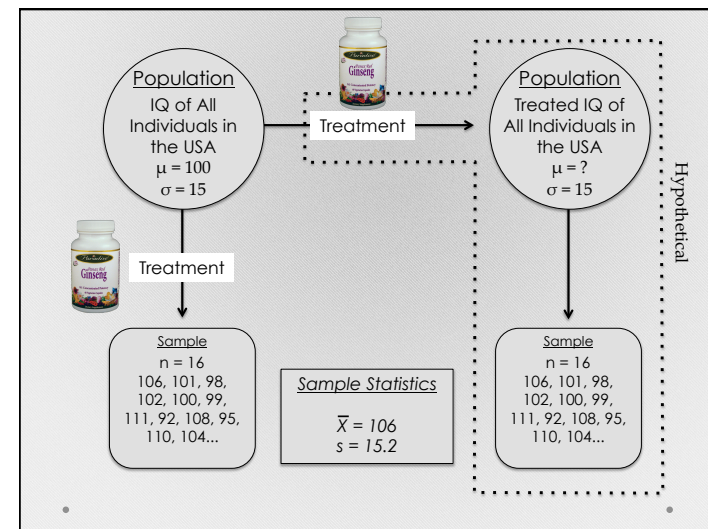
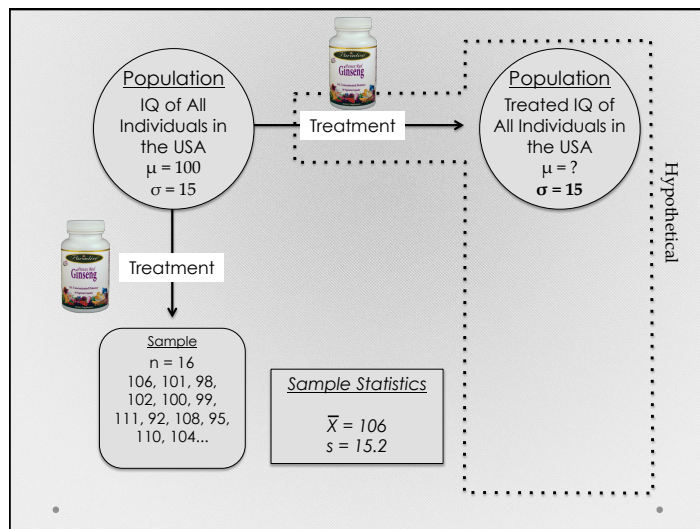
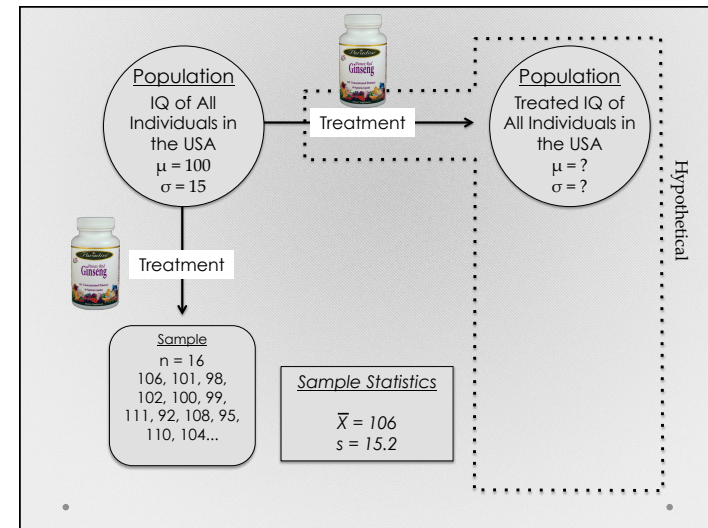
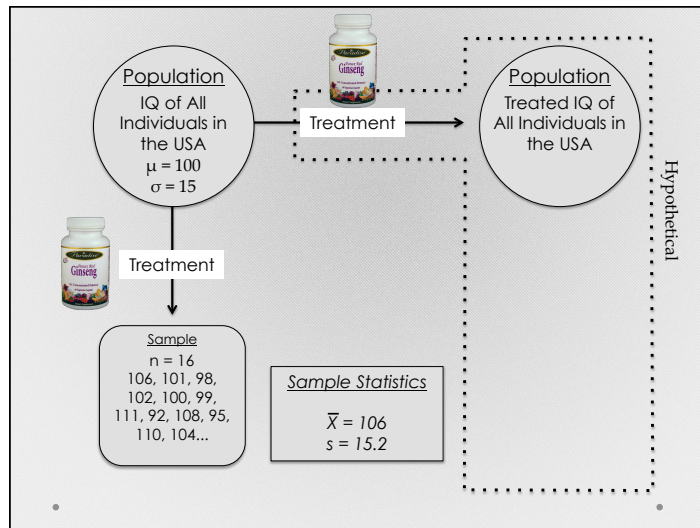
Null Hypothesis ( $H_0$ )

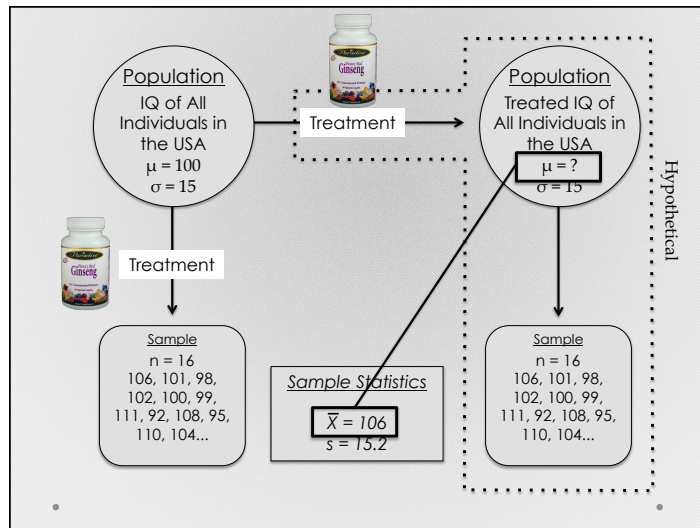
$$\mu_{\text{treatment}} = \mu_{\text{without treatment}}$$

Alternative Hypothesis ( $H_1$ )

$$\mu_{\text{treatment}} \neq \mu_{\text{without treatment}}$$







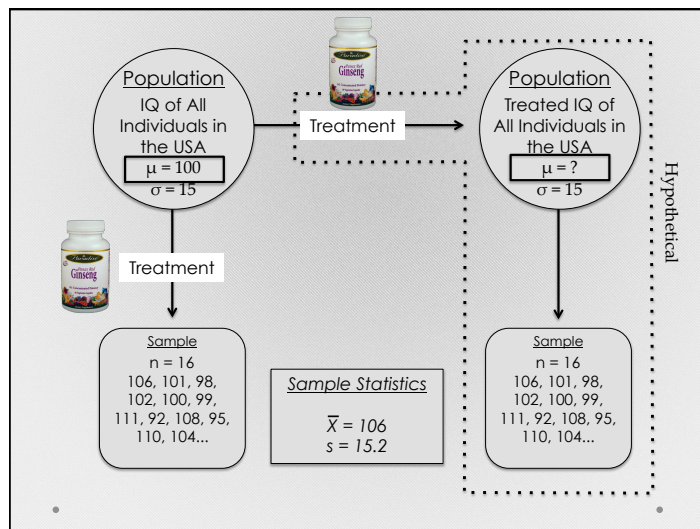
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## Statistical Hypotheses

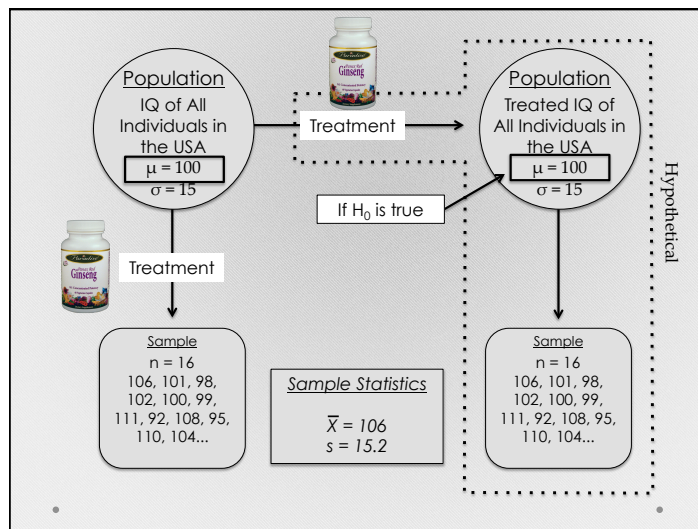
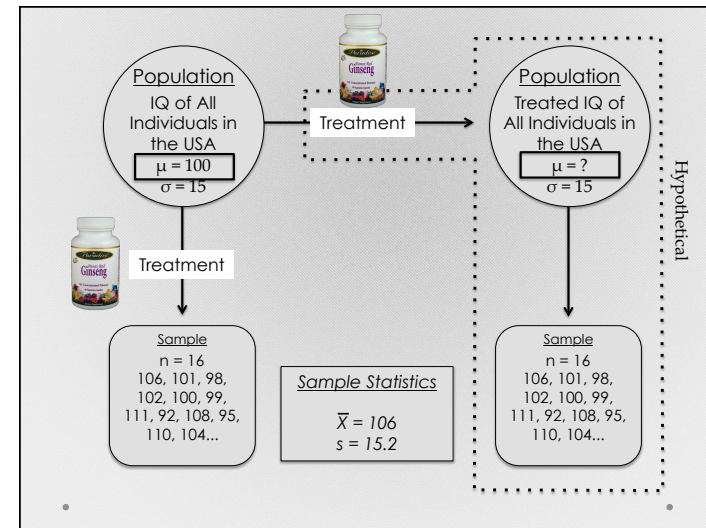
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Statistical hypotheses are always about populations, not samples!



## Statistical Hypotheses

Null Hypothesis ( $H_0$ )

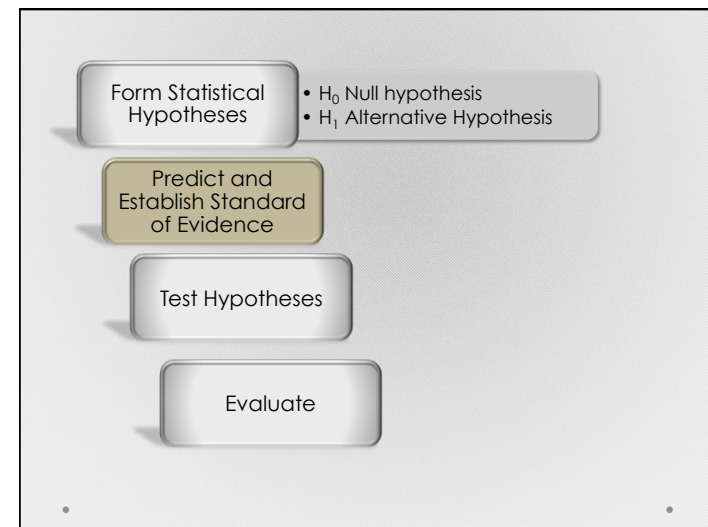
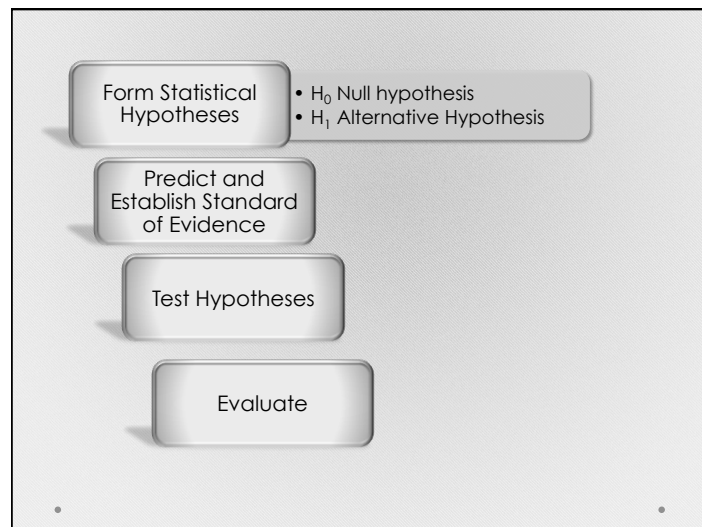
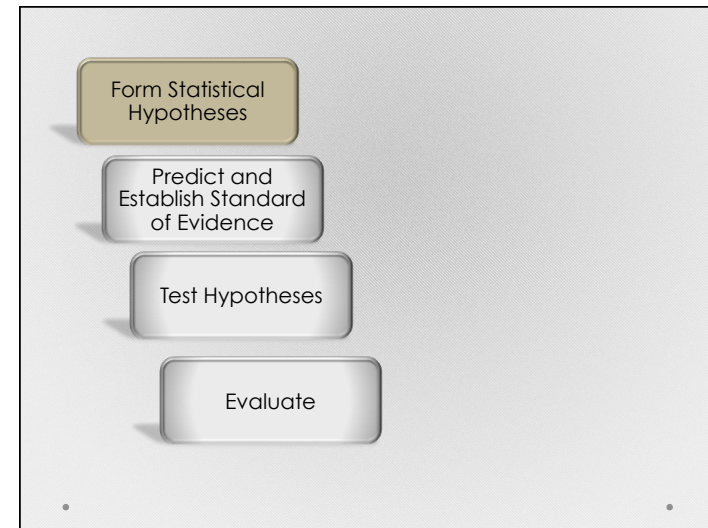
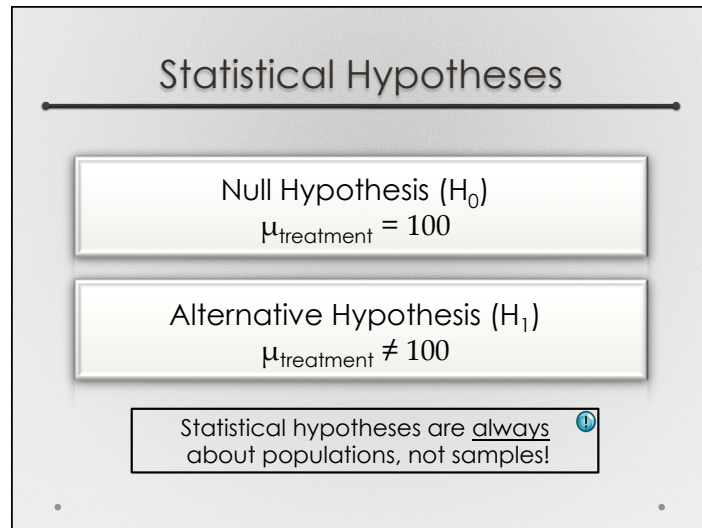
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$$\mu_{\text{treatment}} \neq \mu_{\text{without treatment}}$$

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## Statistical Hypotheses

Null Hypothesis ( $H_0$ )

$$\mu_{\text{treatment}} = 100$$

Alternative Hypothesis ( $H_1$ )

$$\mu_{\text{treatment}} \neq 100$$

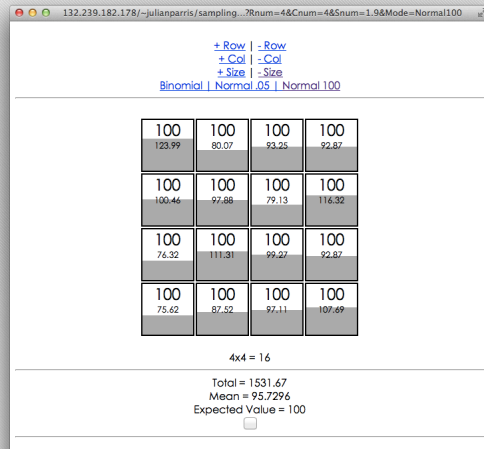
## Statistical Hypotheses

Null Hypothesis ( $H_0$ )

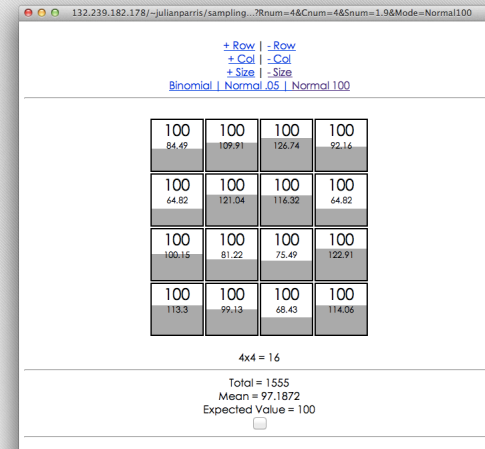
$$\mu_{\text{treatment}} = 100$$

Alternative Hypothesis ( $H_1$ )

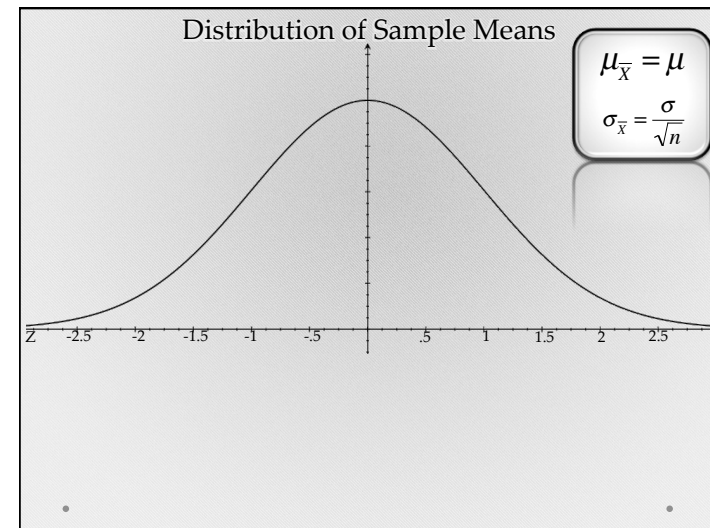
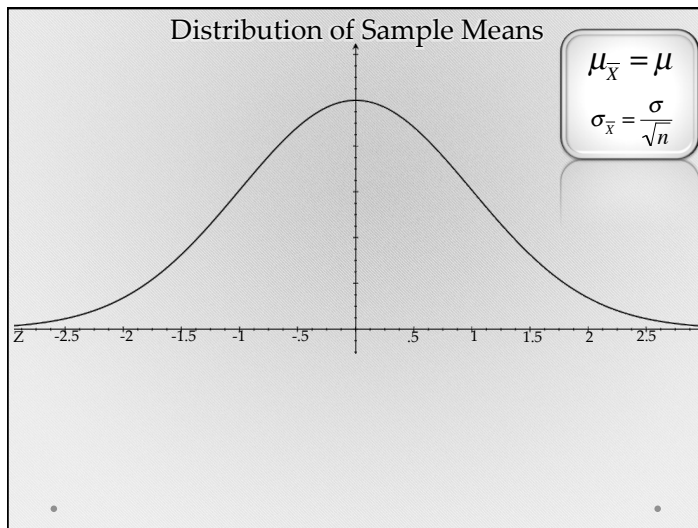
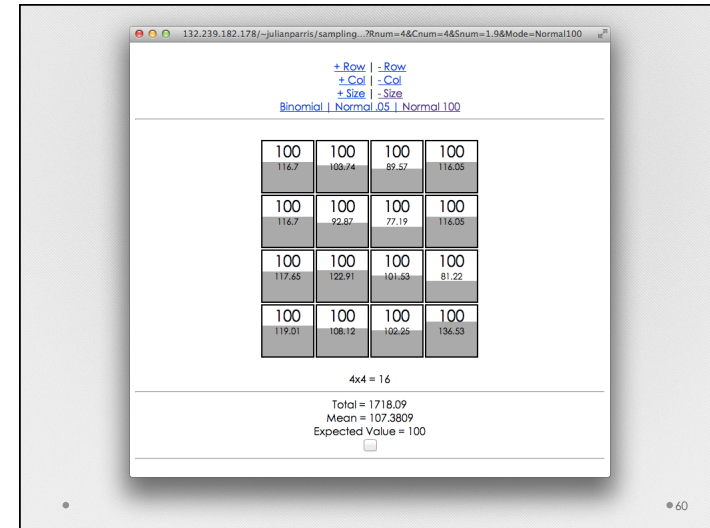
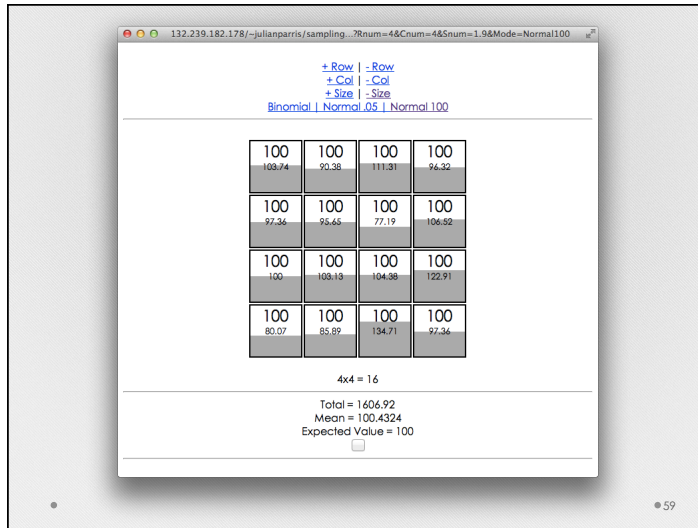
$$\mu_{\text{treatment}} \neq 100$$

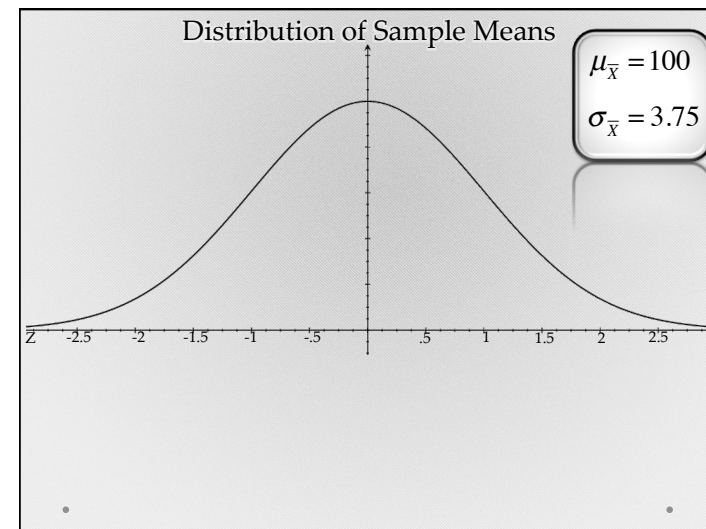
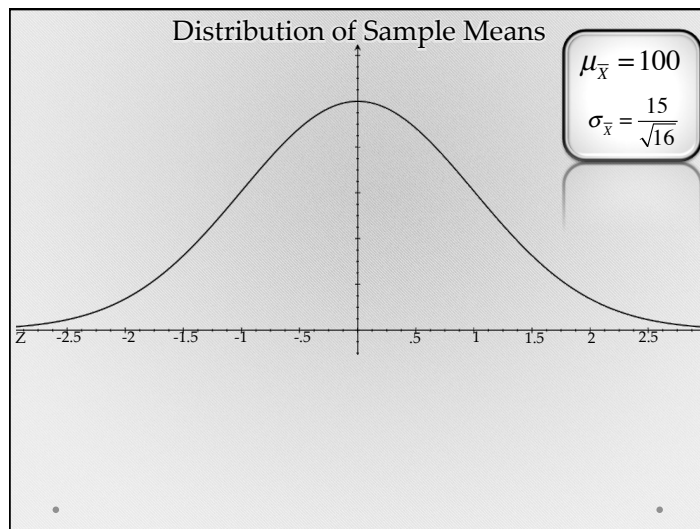
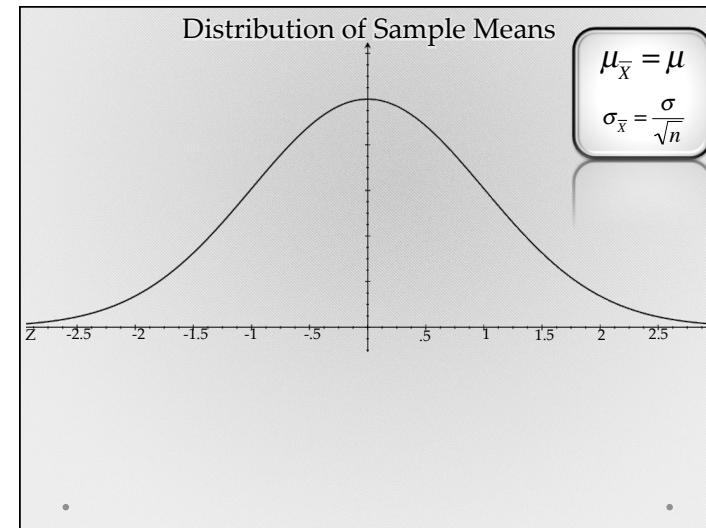
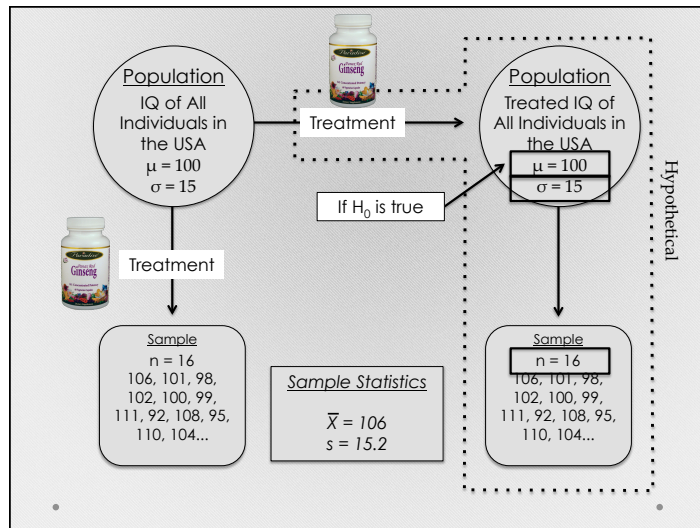


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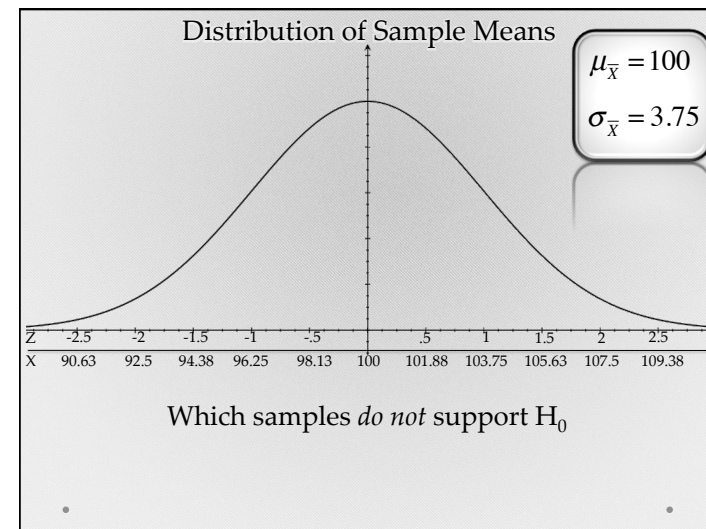
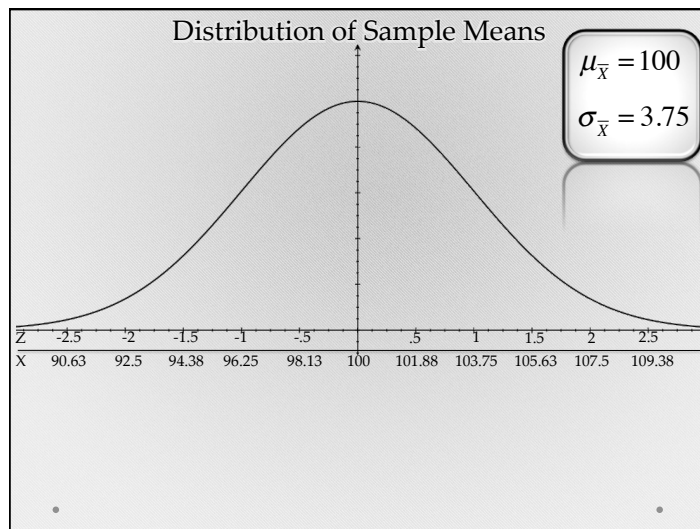
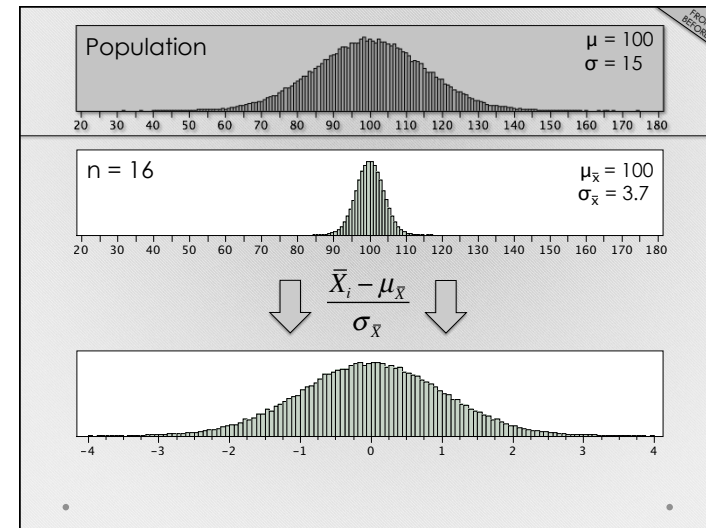
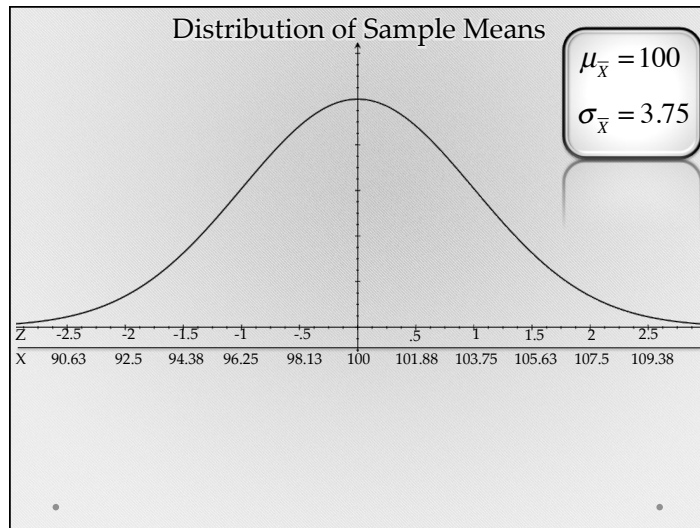


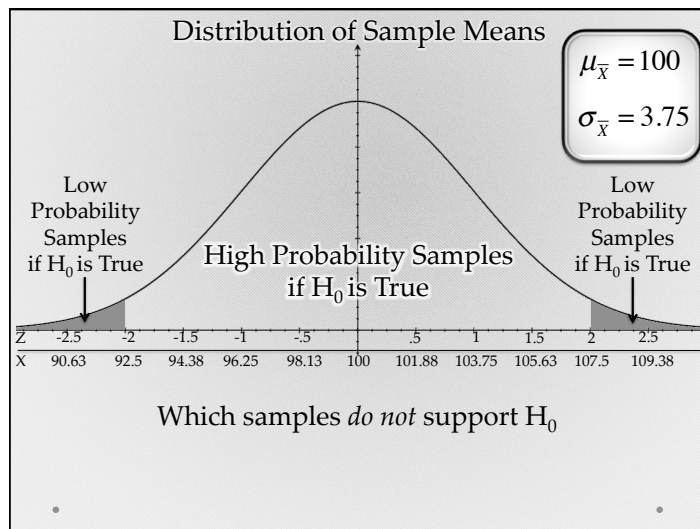
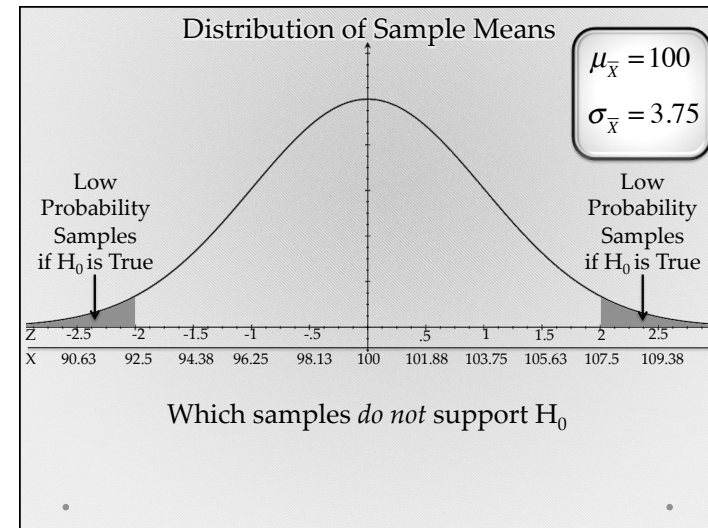
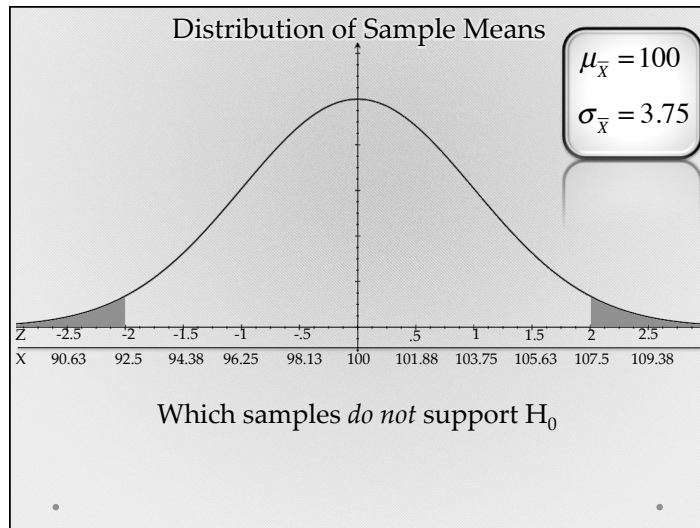
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Alpha Level,  $\alpha$

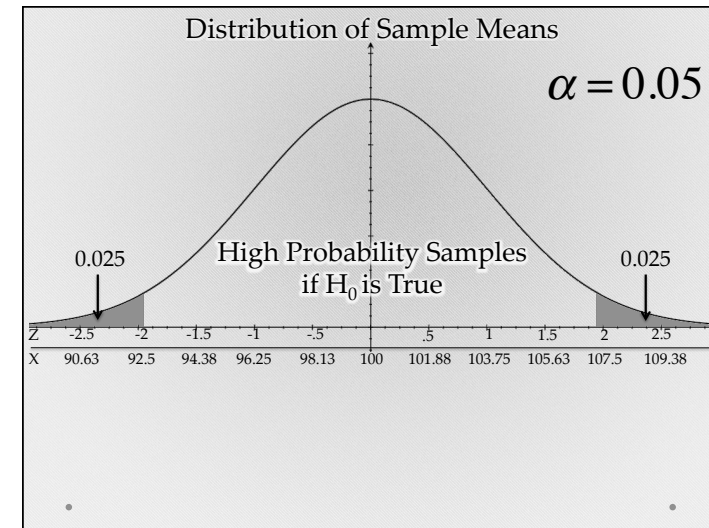
The probability value that is used to define which sample outcomes are considered very unlikely if the null hypothesis is true

## Alpha Level, $\alpha$

The probability value that is used to define which sample outcomes are considered very unlikely if the null hypothesis is true

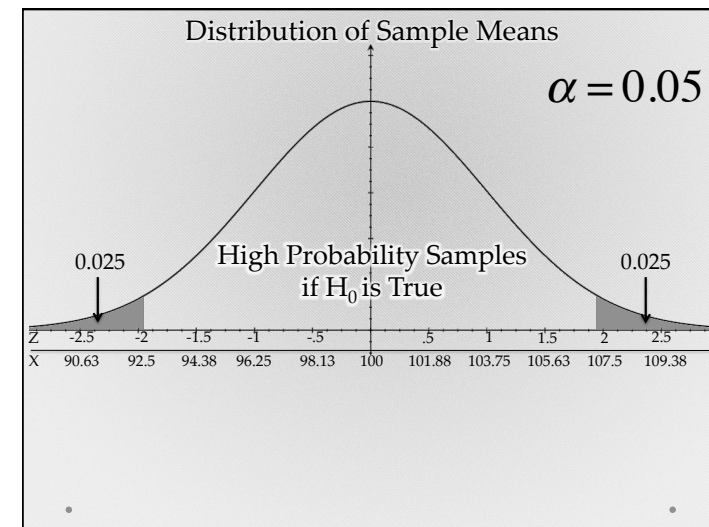
Most Common:

$$\alpha = 0.05$$

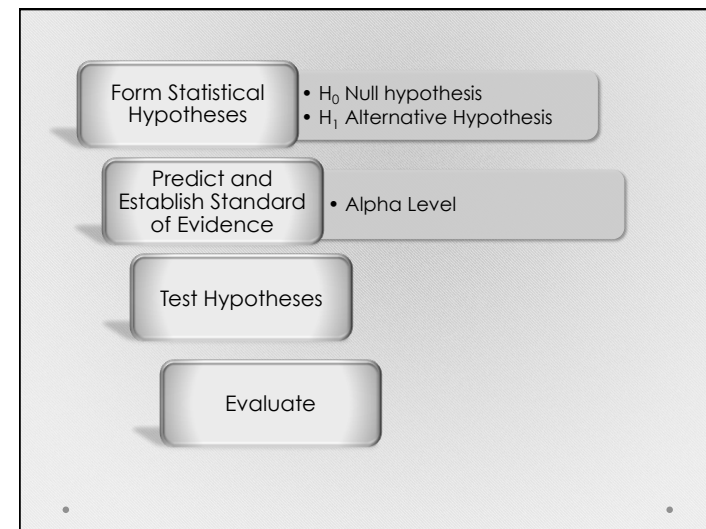
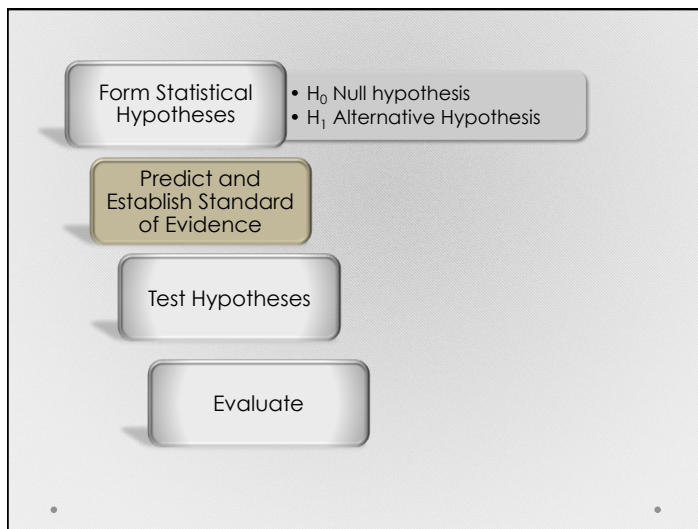
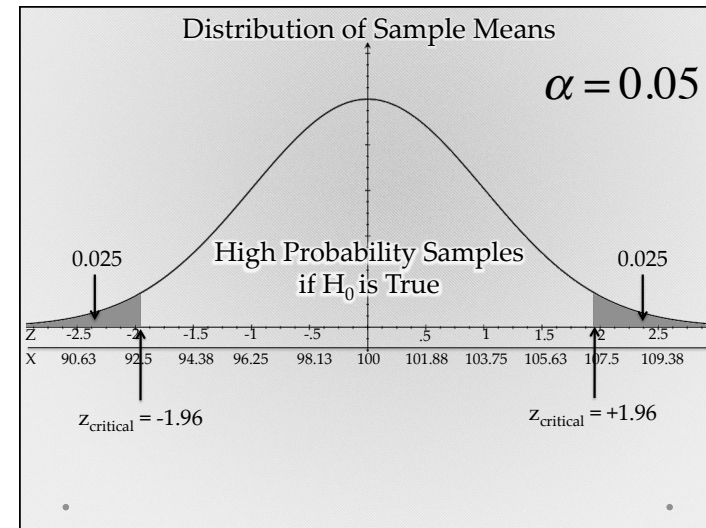
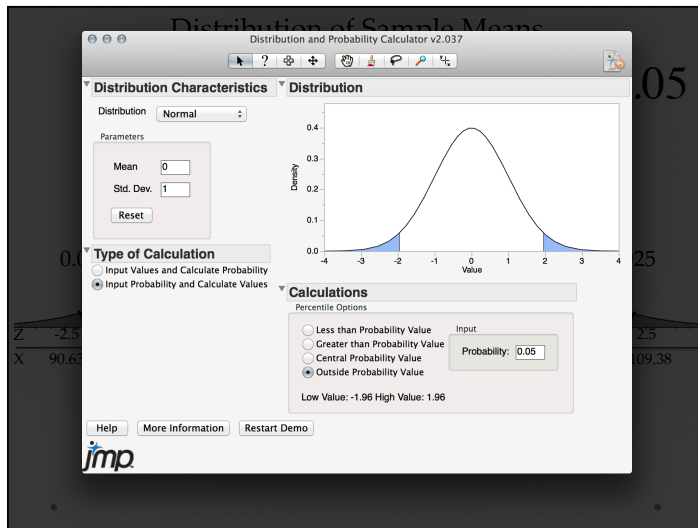


## Critical Region

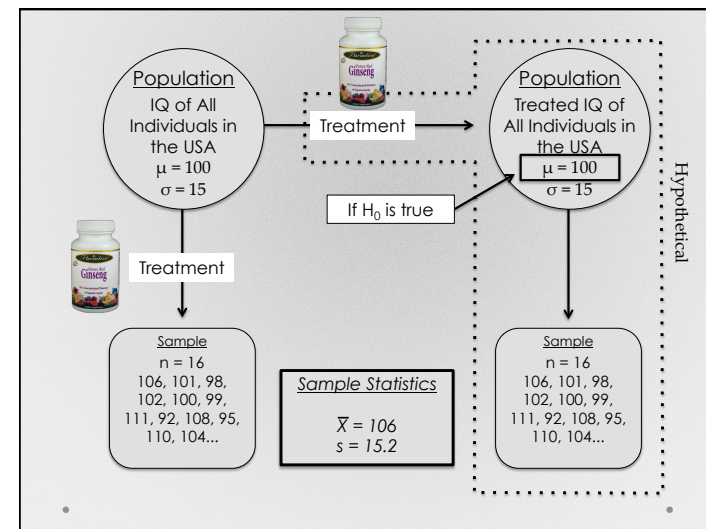
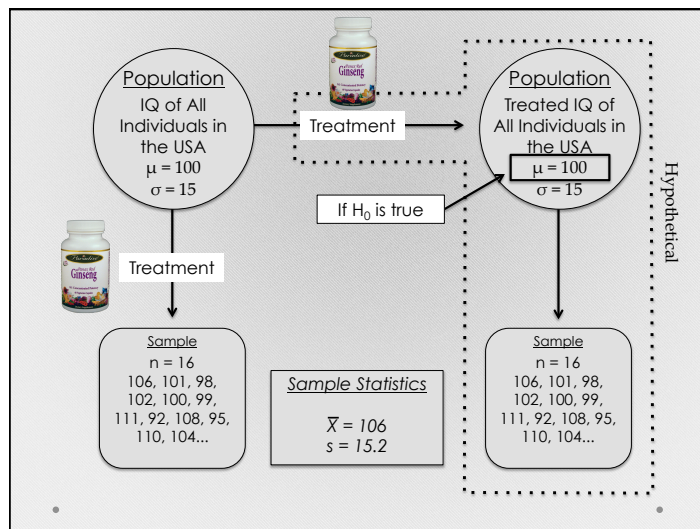
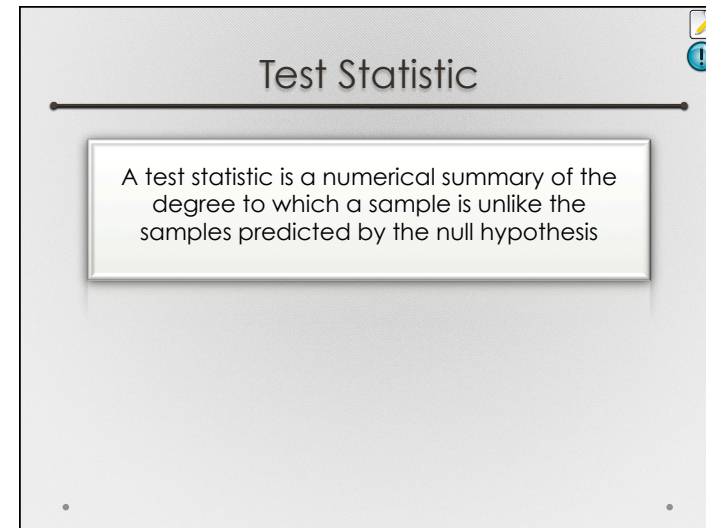
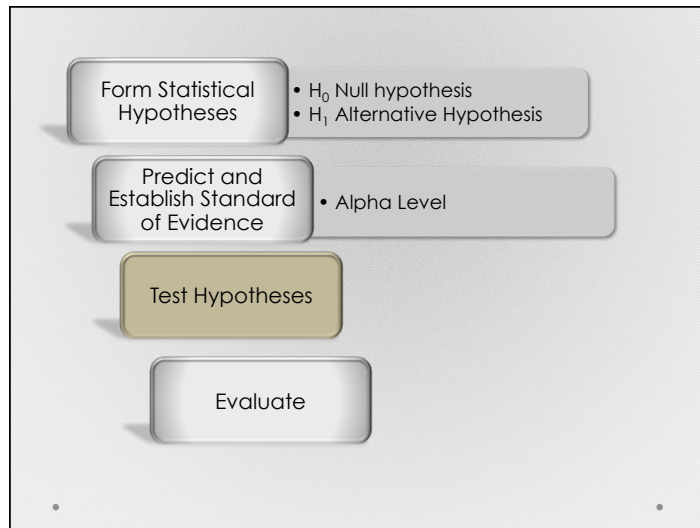
The region of the sampling distribution that contains the sample outcomes that are considered very unlikely if  $H_0$  is true











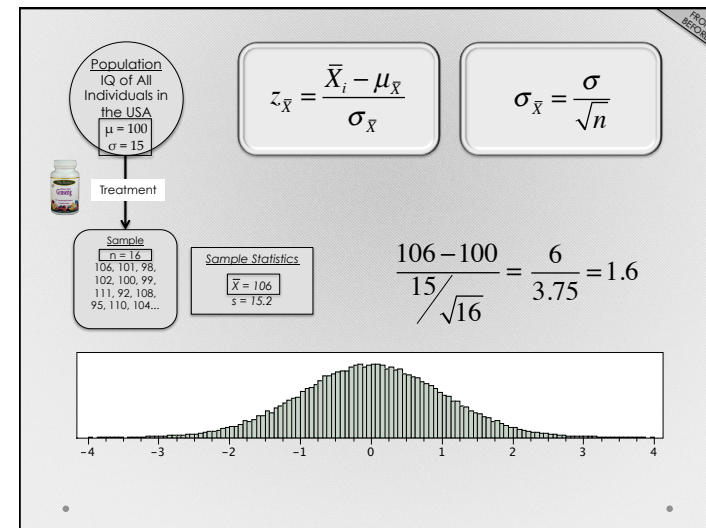
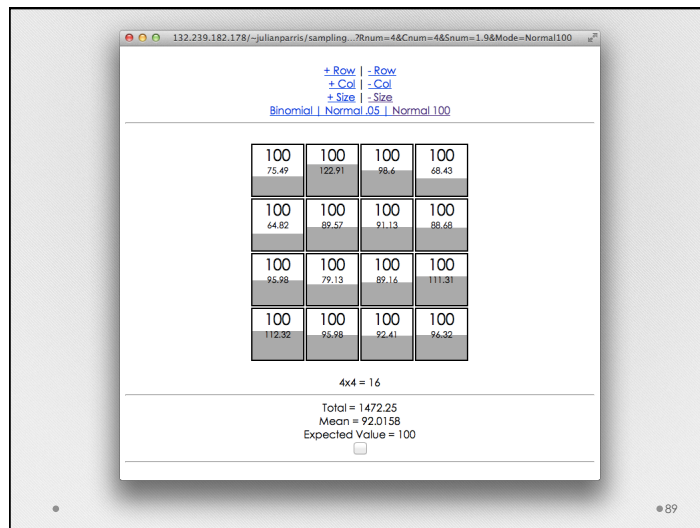
## Z-Test Statistic

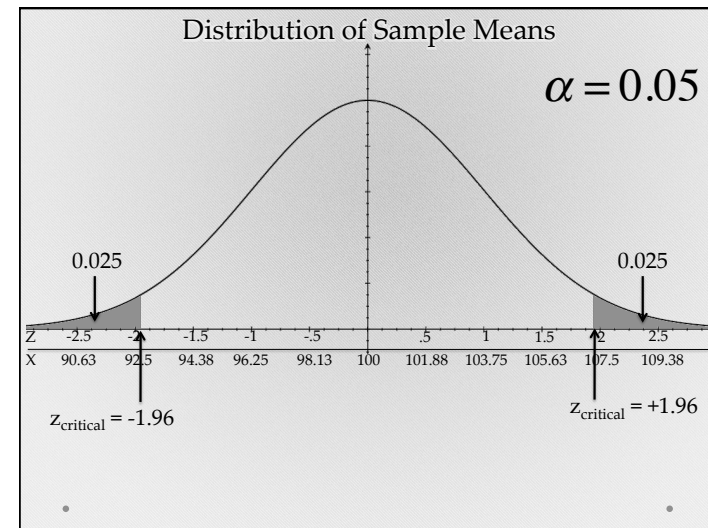
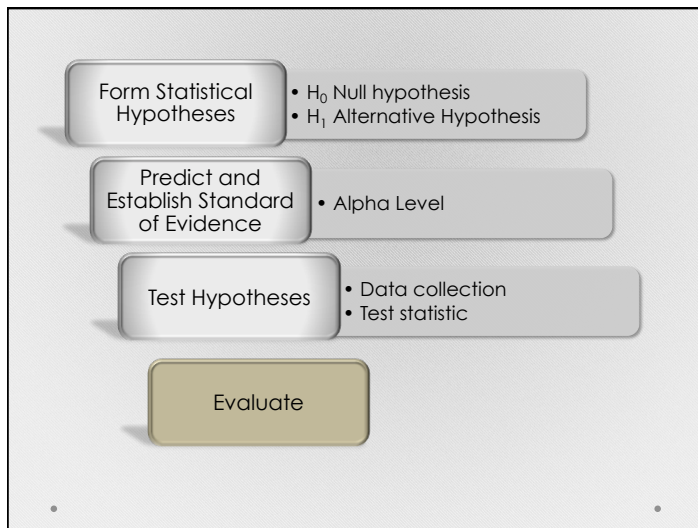
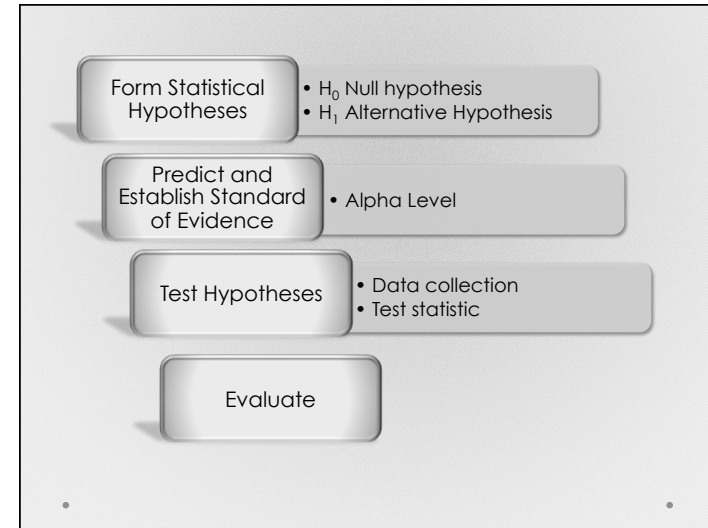
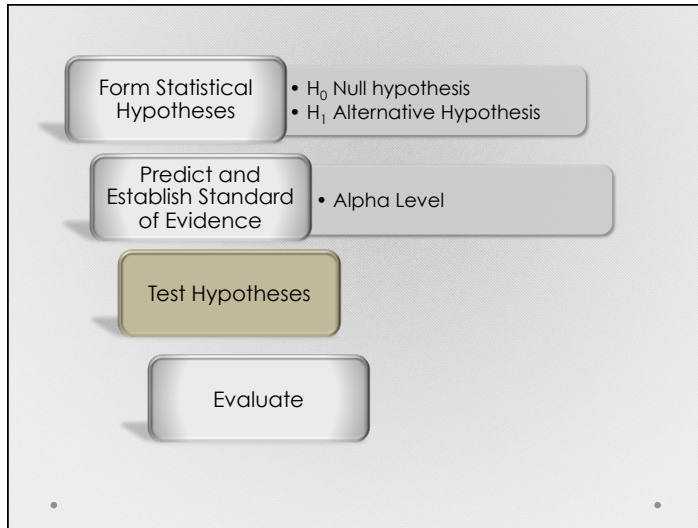
$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

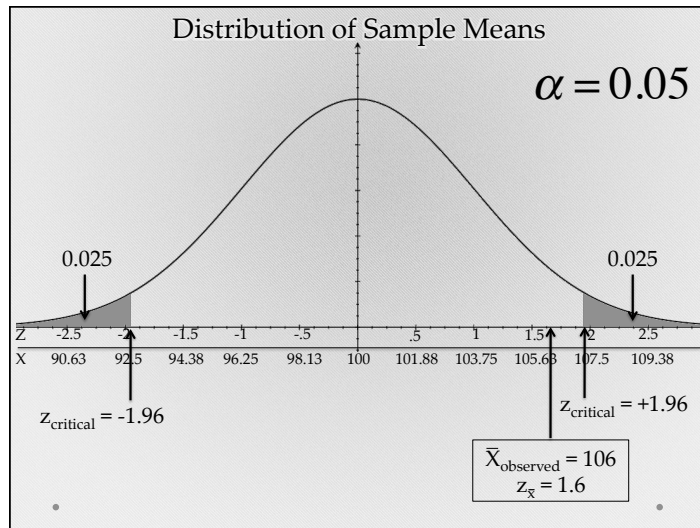
## Z-Test Statistic

$$z_{\bar{X}} = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

value for  $\mu$   
assuming  $H_0$   
is true







## Proportion More Extreme Value

a.k.a. p-value

A p-value is way of describing how extreme a score is in a distribution. A p-value is the proportion of a distribution more extreme than a given score.

One-Tailed P-Value:

Proportion of distribution more extreme in one tail

Two-Tailed P-Value:

Proportion of distribution more extreme in both tails

